CIV 2118 – Introdução ao Método dos Elementos Finitos
2º Semestre – 2009

Trab2: Método dos Elementos Finitos
Elasticidade plana

O objetivo do trabalho é complementar um programa em MATLAB para análise de problemas de elasticidade plana pelo Método dos Elementos Finitos.

Para a complementação do programa é necessário o entendimento do programa incompleto fornecido. Esse entendimento faz parte do trabalho. Os trechos do programa que devem ser complementados são identificados pela linha de comentário do tipo:

```
%%% COMPLETE HERE XX %%%
```

O trabalho deve ser entregue na forma de um relatório, que descreve os procedimentos que foram utilizados para complementar o programa. O código MATLAB completo deve ser enviado via e-mail para os professores. Tal relatório deve conter, para cada trecho complementado, a(s) linha(s) de código MATLAB introduzidas. Por exemplo,

```
%%% COMPLETE HERE 01 %%%
TRECHO DE CÓDIGO MATLAB

%%% COMPLETE HERE 02 %%%
TRECHO DE CÓDIGO MATLAB

...
```

Os arquivos do programa estão disponíveis na *homepage* da disciplina:
http://www.tecgraf.puc-rio.br/~deane/civ2118-092.

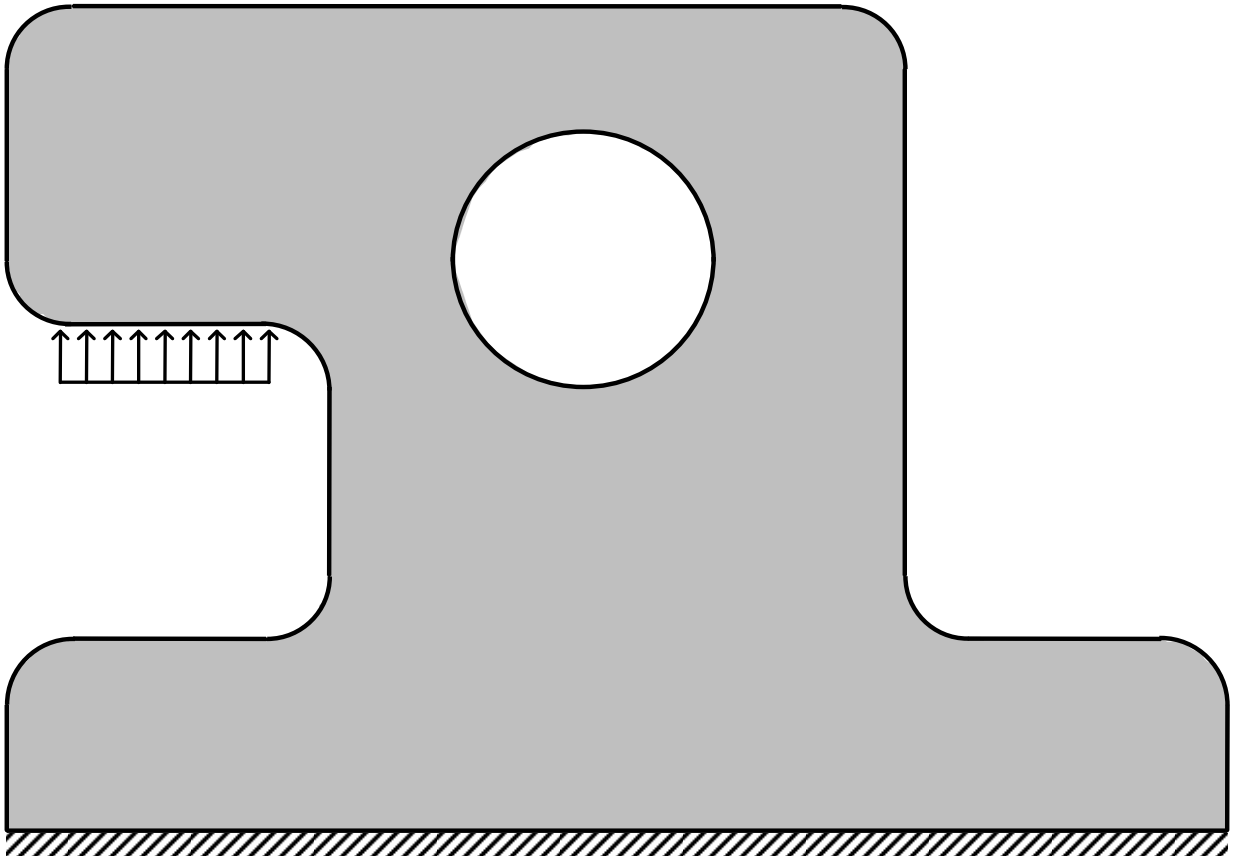
Faça download do arquivo:
ftp://ftp.tecgraf.puc-rio.br/pub/users/lfm/CIV2118-092-trab2.zip.

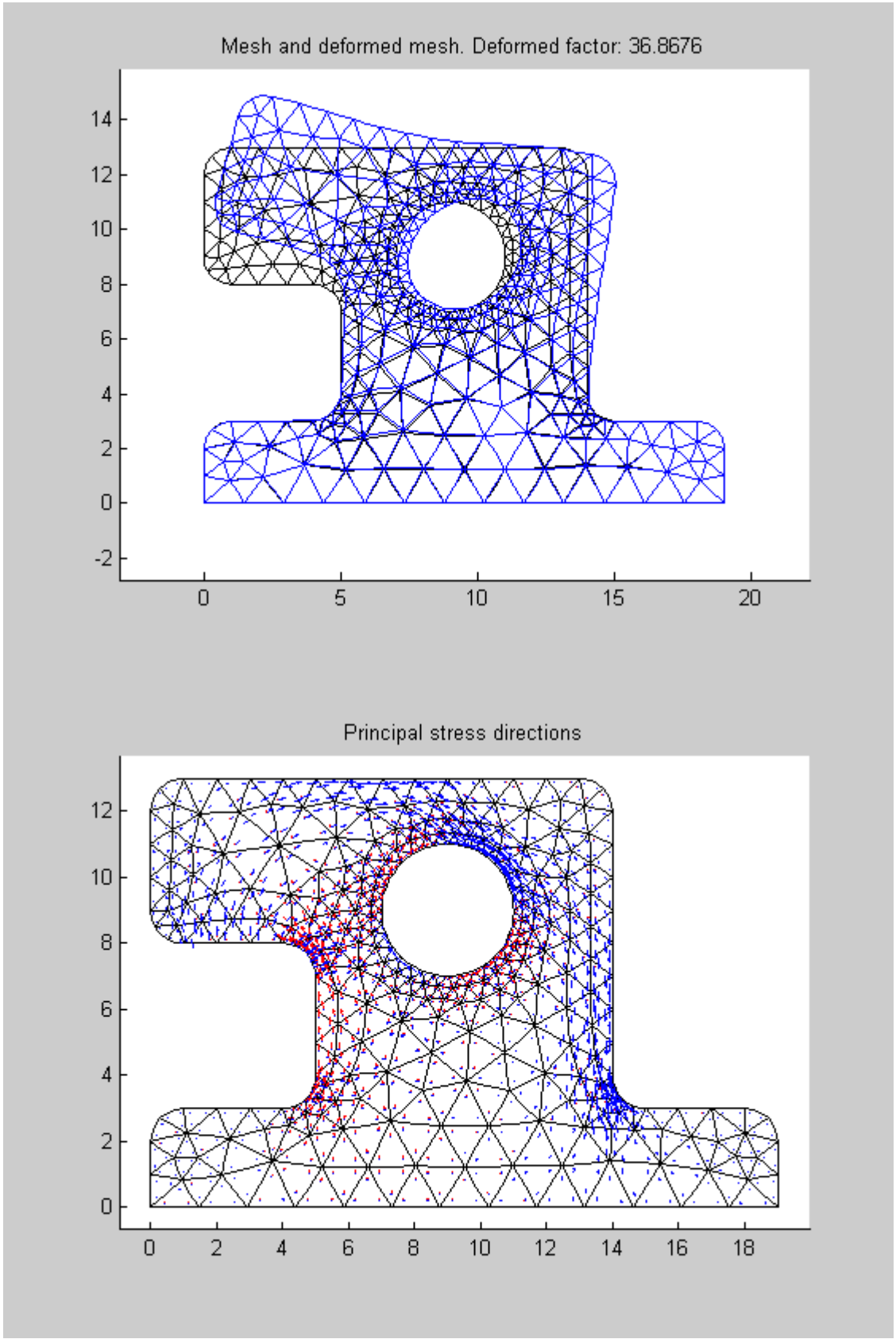
Esse arquivo contém o código fonte (incompleto) de todas as funções MATLAB do programa e alguns arquivos de dados de modelos de elementos finitos em um formato neutro (extensão .nf).

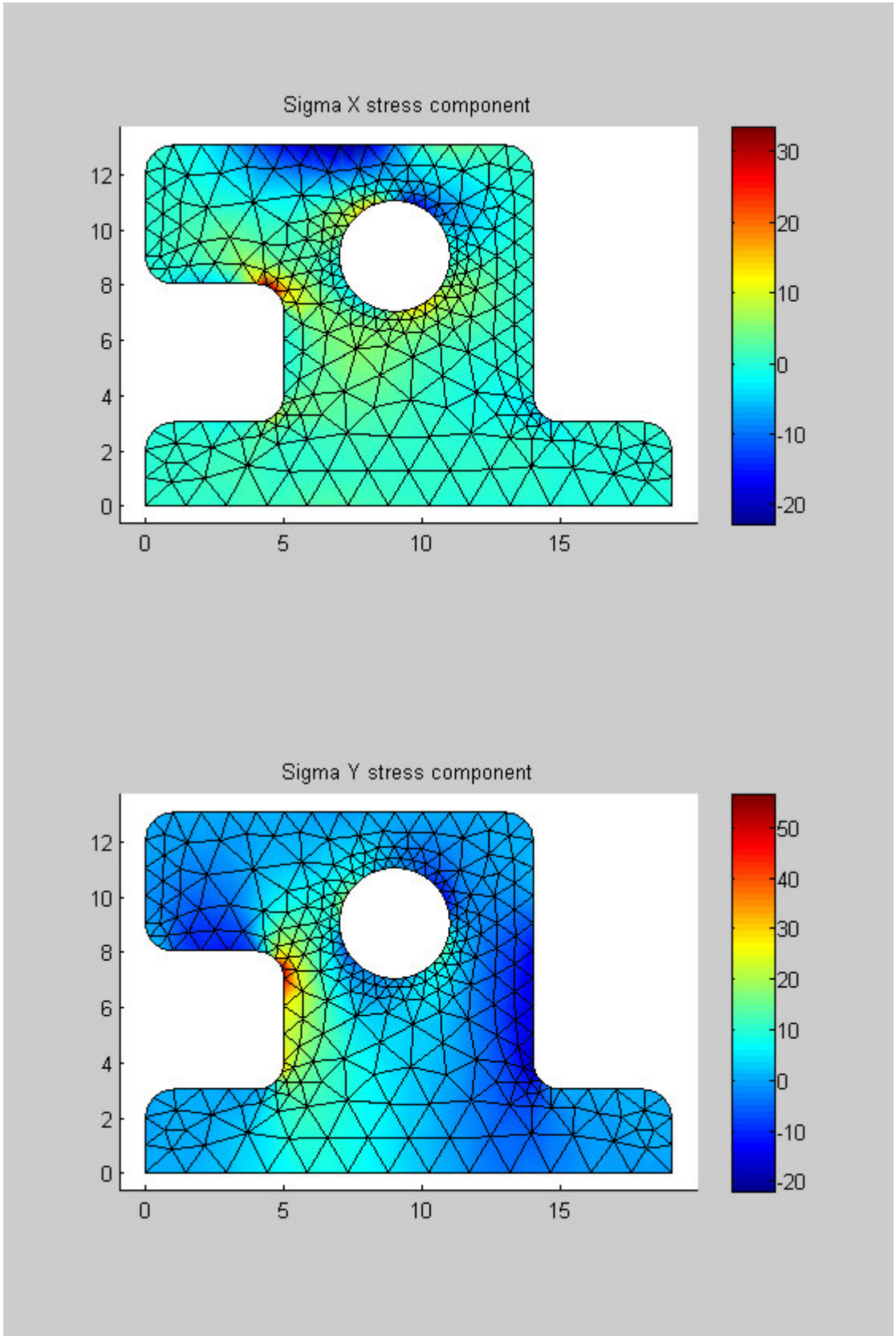
Listagem do arquivo principal do programa (main.m)

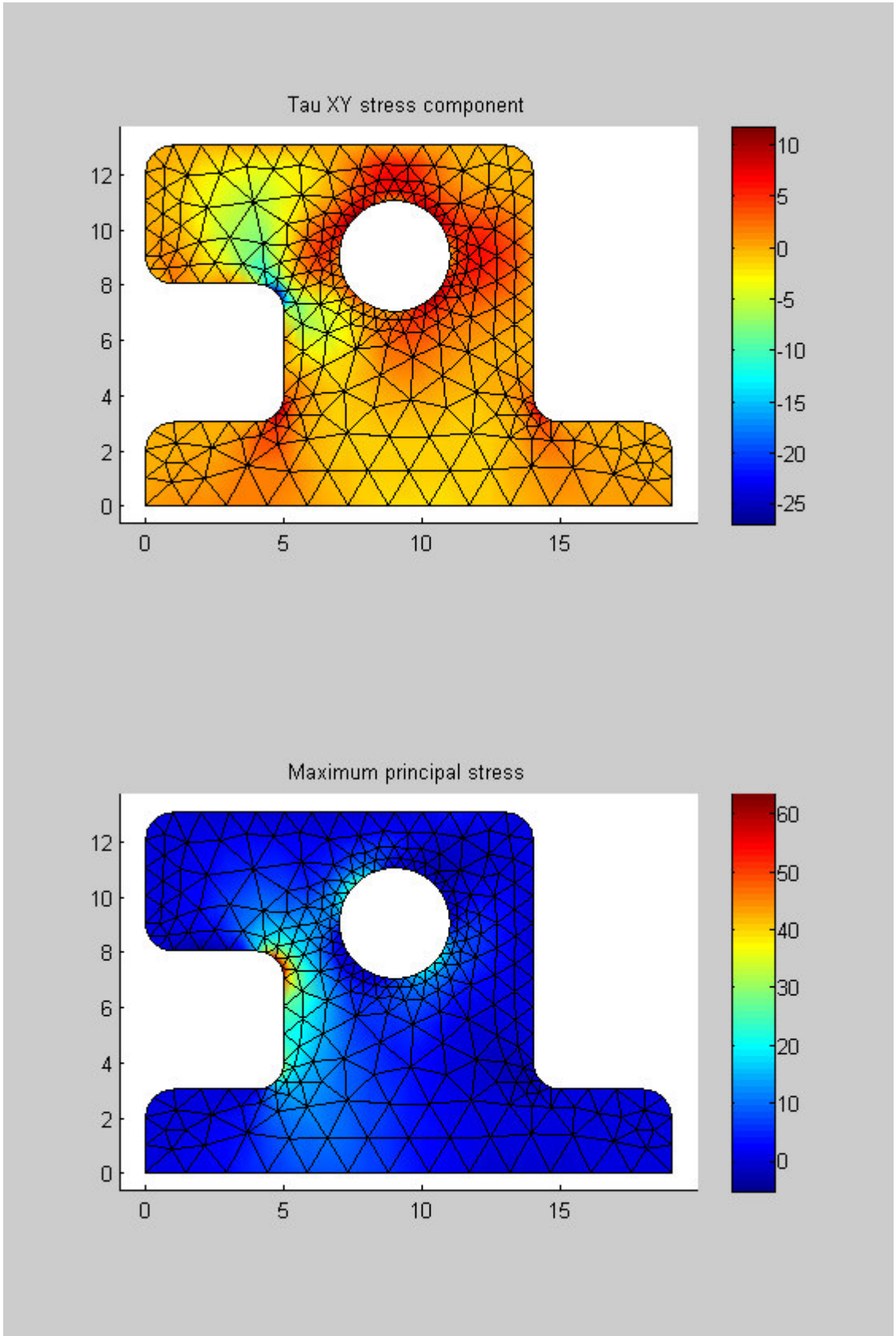
```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% elasticity2D - FEM                                                      %
% Main driver file.                                                       %
% This file contains MATLAB code of a program for linear-elastic,        %
% displacement-based, two-dimensional, finite-element analysis for       %
% solving a stress-distribution elasticity problem.                       %
% The program reads a file with FE model data, in a neutral format,      %
% assembles a system of equations, solves the system and visualizes       %
% the response data.                                                      %
%                                                                         %
% Author:                                                                 %
% Luiz Fernando Martha                                                    %
% Pontifical Catholic University of Rio de Janeiro - PUC-Rio              %
% Department of Civil Engineering and Tecgraf                             %
%                                                                         %
% Adapted from the program Heat2D                                         %
% by Haim Waisman, Rensselaer Polytechnic Institute.                      %
% Available in the the companion site (http://1coursefem.blogspot.com)    %
% of the book by Fish, J. and Belytschko, T., A First Course in Finite    %
% Elements - Chapter 12: Finite Element Programming with MATLAB, 2007.    %
%                                                                         %
% It is assumed that there is only one type of element and only one       %
% type of gauss integration order in each finite element model.           %
%                                                                         %
% It is assumed that there is a single load case.                         %
%                                                                         %
% See global variables in file include_gbl_refs.m.                        %
%                                                                         %
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
clc;
clear all;
close all;

% Include global variables
include_gblrefs;

% Initialize constant global variables
init_constants;

% Preprocessing
[K,F,D] = preProcessor;

% Assemble global stiffness matrix
fprintf(1,'Assembling stiffness matrix...\n');
for e = 1:nel
 ke = elemStiffMtx(e);
 K = drvAssembleMtx(K,e,ke);
end

% Assemble global forcing vector
fprintf(1,'Assembling forcing vector...\n');
F = drvPointLoads(F);    % initialize forcing vector with nodal point loads
F = drvEdgeLoads(F);     % add edge (element side) loads to forcing vector
F = drvAreaLoads(F);     % add area (element) loads to forcing vector

% Partition and solve system of equations
fprintf(1,'Solving system of equations...\n');
[D,F] = drvSolveEqnSystem(neq,neqfree,K,F,D);

% Postprocessing
posProcessor(D);

fprintf(1,'Finished.\n');
```

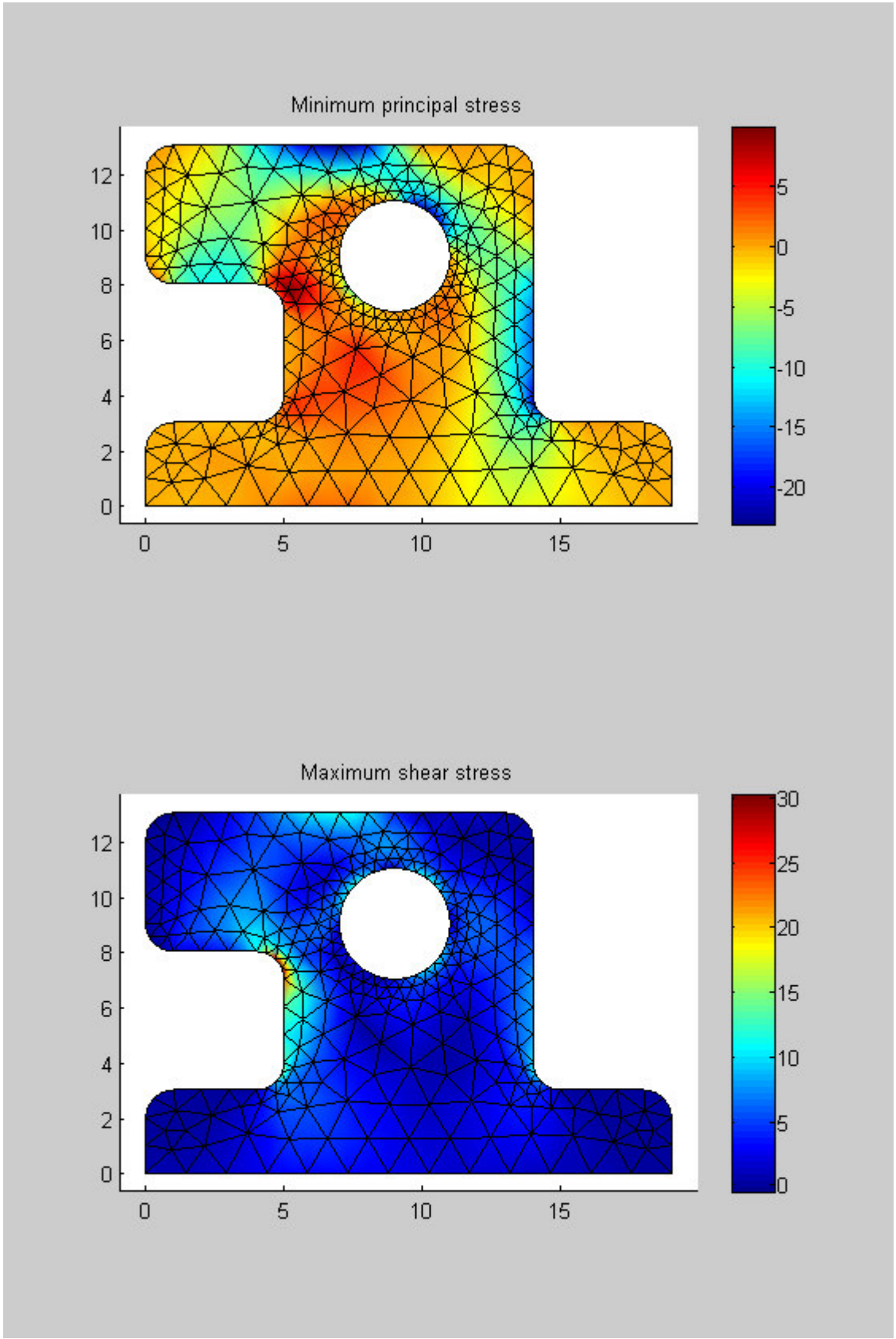
Resultados do exemplo LatchT6.nf



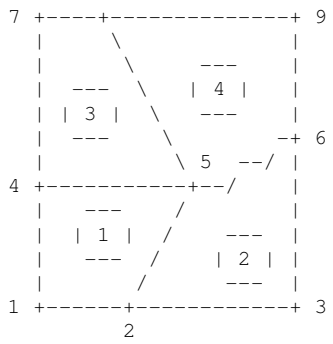

Mesh and deformed mesh. Deformed factor: 36.8676
14
12
10
8
6
4
2
0
-2
0
5
10
15
20
Principal stress directions
12
10
8
6
4
2
0
0
2
4
6
8
10
12
14
16
18


Sigma X stress component
12
10
8
6
4
2
0
0
5
10
15
30
20
10
0
-10
-20
Sigma Y stress component
12
10
8
6
4
2
0
0
5
10
15
50
40
30
20
10
0
-10
-20


Tau XY stress component
12
10
8
6
4
2
0
0
5
10
15
10
5
0
-5
-10
-15
-20
-25
Maximum principal stress
12
10
8
6
4
2
0
0
5
10
15
60
50
40
30
20
10
0


Minimum principal stress
12
10
8
6
4
2
0
0
5
10
15
5
0
-5
-10
-15
-20
Maximum shear stress
12
10
8
6
4
2
0
0
5
10
15
30
25
20
15
10
5
0

Descrição dos arquivos do programa Elasticity2D

main	**Main driver and global variable files**
main.m	Main driver file. This file contains MATLAB code of a program for linear-elastic, displacement-based, two-dimensional, finite-element analysis for solving a stress-distribution elasticity problem. The program reads a file with FE model data, in a neutral format, assembles a system of equations, solves the system and visualizes the response data.
include_gblrefs.m	File to include global variables
init_constants.m	Initialize constant global variables.
pre	**Pre-processing files**
preProcessor.m	Preprocessing input data and sets up mesh information. Output arguments: K: global stiffness matrix (returns initialized with null values) F: global forcing vector (returns initialized with null values) D: global displacement vector (returns initialized with null values and with known support settlement values)
input_patchtest_q4.m	Input data for patch test with 4 QUAD4 elements in a irregular mesh. There are prescribed displacaments in x direction and applied tractions in y direction. 
preReadNF.m	This fuction reads a neutral-format file with input data for a finite- element model for this program. The neutral format is described in www.tecgraf.puc-rio.br/neutralfile. It is assumed that all nodes and elements are numbered consecutively from one to the total number of nodes and elements. It is assumed that there is only one type of element and only one type of gauss integration order in each finite element model. It is assumed that there is a single load case.
node	**Node files**
nodeSetupDOFNum.m	Setup global d.o.f numbering matrix. Free d.o.f.'s are numbered first. Counts total number of equations of free d.o.f.'s and total number of equations of fixed d.o.f.'s, and stores in global variables. Incomplete file. Need to complete in: %%% COMPLETE HERE 01 %%% %%% COMPLETE HERE 02 %%%
shp	**Finite-element shape files**
shpNodeCCWOrder.m	Returns the resequence vector of nodal indices of an element in CCW order. Output arguments: rv: resequence vector of nodal indices of an element in CCW order
shpNodeParCoords2D.m	Returns the parametric coordinates of the nodal points of an element. Output arguments: pt: array with pairs of element nodal paramatric coordinates

shpNmatrix2D.m	Evaluates 2D shape function matrix. Evaluates shape functions (in parametric coordinates) at point (r,s) for 2D elements. Input arguments: r: first parametric coordinate value s: second parametric coordinate value Output arguments: N: shape function matrix evaluated at given position Incomplete file. Need to complete in: %%% COMPLETE HERE 03 %%% %%% COMPLETE HERE 04 %%% %%% COMPLETE HERE 05 %%%
shpGradNmatrix2D.m	Evaluates derivatives of shape functions (in parametric coordinates). Input arguments: r,s: parametric coordinate values Output arguments: GradN: Shape function gradient matrix evaluated at given position Incomplete file. Need to complete in: %%% COMPLETE HERE 06 %%% %%% COMPLETE HERE 07 %%% %%% COMPLETE HERE 08 %%%
shpDetJ2D.m	Evaluates 2D Jacobian determinant. Evaluates determinant of Jacobian matrix of parametric to cartesian transformation at point (r,s) in parametric coordinates. Input arguments: r,s: parametric coordinate values C: nodal coordinate matrix Output arguments: detJ: determinant of Jacobian matrix evaluated at given position Incomplete file. Need to complete in: %%% COMPLETE HERE 09 %%%
shpGetEdge.m	Get nodal incidence on a given 2D element edge (side). Input arguments: e: element number nod1: first node on element edge nod2: last node on element edge Output arguments: nne: number of nodes of target element edge IENe: local nodal connectivity array of target element edge
shpNmatrix1D.m	Evaluates 1D shape function matrix. Evaluates shape functions (in parametric coordinate) at point (r) for 1D elements. Input arguments: r: parametric coordinate value Output arguments: N: shape function matrix evaluated at given position
shpDetJ1D.m	Evaluates 1D (edge) Jacobian determinant. Evaluates determinant of Jacobian matrix of parametric to cartesian transformation at point (r) in parametric coordinate. Input arguments: r: parametric coordinate value C: nodal coordinate matrix Output arguments: detJ: determinant of Jacobian matrix evaluated at given position
gauss	**Gauss quadrature files**
gauss.m	Get gauss points coordinates in the parametric space of element and corresponding weights for a given quadrature type and order. Input arguments: type: gauss quadrature type (either line, triangle or quadrilateral) order: quadrature order Output arguments: ngp: number of gauss points w: array of gauss point weights gp: array of gauss point parametric coordinates Incomplete file. Need to complete in: %%% COMPLETE HERE 10 %%% %%% COMPLETE HERE 11 %%% %%% COMPLETE HERE 12 %%% %%% COMPLETE HERE 13 %%%

anm	**Analysis model files**
anmEMtx.m	Generate material constituive matrix for a given element. The material constitutive matrix is assembled using the element modulus of elasticity and poisson ratio. The matrix size and layout depends on the type of analysis. For plane stress and plane strain analysis, the size is (3,3). For axisymmetric analysis, the size is (4,4). Input arguments: e: index of element Incomplete file. Need to complete in: %%% COMPLETE HERE 14 %%% %%% COMPLETE HERE 15 %%% %%% COMPLETE HERE 16 %%%
anmBMtx.m	Assemble 2D B matrix for an element. Evaluates derivatives of shape functions (in physical Cartesian coordinates) at point (r,s) in parametric coordinates and assembles them in the B matrix. In case of axisymmetric analysis, the shape function itself is also used. Input arguments: r,s: parametric coordinate values C: nodal coordinate matrix Output arguments: B: B matrix evaluated at given position Incomplete file. Need to complete in: %%% COMPLETE HERE 17 %%% %%% COMPLETE HERE 18 %%% %%% COMPLETE HERE 19 %%%
anmPointStress.m	Get stress components (sigma x, sigma y and tau xy) at a given point on an element. For axisymmetric analysis the sigma z stress component is desregarded. Input arguments: r,s: parametric coordinate values of point location C: nodal coordinate matrix of target element E: constituive matrix of target element d: generalized displacements/rotations for all d.o.f.'s of element Output arguments: str: stress components (sx,sy,txy) at target point Incomplete file. Need to complete in: %%% COMPLETE HERE 20 %%%
elem	**Finite element files**
elemStiffMtx.m	Generate stiffness matrix for a given element. Input arguments: e: index of target element Incomplete file. Need to complete in: %%% COMPLETE HERE 21 %%%
elemAreaENL.m	Generate equivalent nodal load vector (feq) for an area uniform distributed load (qx,qy) on a given element. Input arguments: e: element number q: area uniform distributed load values: q(ndof,1) Output arguments: nne: number of nodes of equivalent nodal load vector IENe: local nodal connectivity array of equivalent nodal load feq: equivalent nodal load vector Incomplete file. Need to complete in: %%% COMPLETE HERE 22 %%%
elemEdgeENL.m	Generate equivalent nodal load vector (feq) for an edge uniform distributed load (px,py) on a given element side. Input arguments: e: element number nod1: first node on element edge nod2: last node on element edge p: edge uniform distributed load values: p(ndof,1) Output arguments: nne: number of nodes of equivalent nodal load vector IENe: local nodal connectivity array of equivalent nodal load feq: equivalent nodal load vector Incomplete file. Need to complete in:

	%%% COMPLETE HERE 23 %%%
elemGaussStress.m	Get stress components at gauss points and gauss point cartesian coordinates for a given element. Input arguments: D: solution vector: generalized displacements/rotations for all d.o.f.'s e: target finite element index Output arguments: ngp: number of gauss points for stress evaluation str: stress components (sx,sy,txy) at each gauss point gpc: gauss point cartesian coordinates array Incomplete file. Need to complete in: %%% COMPLETE HERE 24 %%%
elemTRMtx.m	Compute gauss-to-node results transfer matrix (TR matrix). Refs.: Hinton & Campbell, "Local and Global Smoothing of Discontinous Finite Element Functions using a Least Squares Method", Int. J. Num. Meth. Engng., Vol. 8, pp. 461-480, 1974. Burnett, D.S., "Finite Element Analysis - From Concepts to Applications", Addison-Wesley, 1987. Martha, L.F., "Notas de Aula do Curso CIV 2118 - Metodo dos Elementos Finitos", 1994. This functions loads global variables TR and n_gaussstress_pts (see file include_gblrefs.m).
drv	**Analysis driver files**
drvAssembleMtx.m	Assemble global stiffness matrix. Input/output arguments: K: global stiffness matrix Input arguments: e: element number ke: element local stiffness matrix Incomplete file. Need to complete in: %%% COMPLETE HERE 25 %%%
drvPointLoads.m	Add point (nodal) loads to global forcing vector. Add nodal load components to any term of the global forcing vector, incluing the terms that correspond to constrained d.o.f. Input/output arguments: F: global forcing vector Incomplete file. Need to complete in: %%% COMPLETE HERE 26 %%% %%% COMPLETE HERE 27 %%%
drvAreaLoads.m	Add area (element) equivalent nodal loads to global forcing vector. Input/output arguments: F: global forcing vector
drvEdgeLoads.m	Add edge (element side) equivalent nodal loads to global forcing vector. Input/output arguments: F: global forcing vector
drvAssembleENL.m	Add equivalent nodal load vector to the global forcing vector. Assemble the given equivalent nodal load vector to any term of the global forcing vector, incluing the terms that correspond to constrained d.o.f. Input/output arguments: F: global forcing vector Input arguments: nne: number of nodes of given equivalent nodal load vector IENe: array of nodes of given equivalent nodal load feq: equivalent nodal load vector Incomplete file. Need to complete in: %%% COMPLETE HERE 28 %%% %%% COMPLETE HERE 29 %%%
drvSolveEqnSystem.m	Partition and solve the system of equations. Input arguments: neq: total number of equations neqfree: number of equations of free d.o.f.'s K: global stiffness matrix Input/Output arguments:

	F: global force vector (right hand side) D: global displacement vector
pos	**Post-processing files**
posProcessor.m	Visualize mesh and results of current analysis. Input arguments: D: global displacement vector
posPrincStress.m	Get principal stress components and orientation for a given stress tensor. Input arguments: str: stress tensor (sx, sy, txy) stored in a column vector. Output arguments: prc: principal stress components (s1, s2, taumax) stored in a column vector. thetap: angle of normal of principal stress plane w.r.t x axis (angle is returned in radians from 0 to 180 degrees).
posGaussStresses.m	Create arrays and compute gauss stress components and principal stresses for all elements. Input arguments: D: solution vector: generalized displacements/rotations for all d.o.f.'s All gauss point location and stress arrays are declared as global variables (see file incinclude_gblrefs.m).
posElemStressesExtrap.m	Create arrays and compute node extrapolated stress components and principal stresses for all elements. The nodal stress components are computed by extrapolation of gauss point stress components using the TR matrix (which is a global variable that is assumed already created). All stress arrays are declared as global variables (see file include_gblrefs.m).
posNodeStressesExtrap.m	Create arrays and compute extrapolated node smoothed stress components and principal stresses for all nodes. The nodal stress components are computed by averaging values of element extrapolated nodal stress components of all elements adjacent to each node. All stress arrays are declared as global variables (see file include_gblrefs.m).
posCreateFigs.m	Creates figures for post-processing results. Eight windows are created and positioned on the screen. Each window plots a type of post-process result.
posPlotMesh.m	Plots mesh in current active figure.
posPlotDeformMesh.m	Plots deformed mesh in current active figure.
posPlotNodeContour.m	Plots current active node contour response in current active figure.