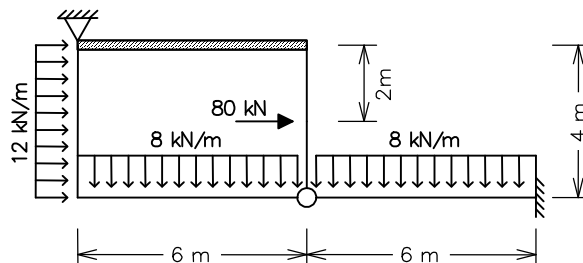


# CIV 1127 – ANÁLISE DE ESTRUTURAS II – 2º Semestre – 2007

## Segunda Prova – 29/10/2007 – Duração: 2:45 hs – Sem Consulta

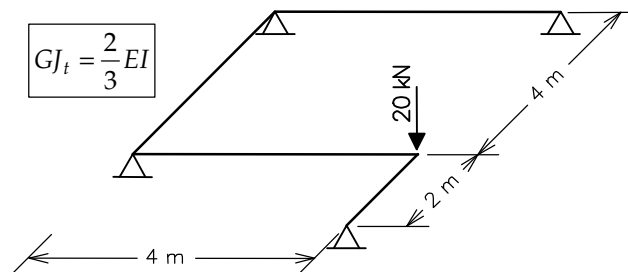
### 1ª Questão (5,5 pontos)

Empregando-se o Método dos Deslocamentos, obter o diagrama de momentos fletores para o quadro ao lado (barras inextensíveis). Todas as barras têm a mesma inércia à flexão  $EI = 2.4 \times 10^4 \text{ kNm}^2$ , com exceção da barra horizontal superior que é infinitamente rígida à flexão.



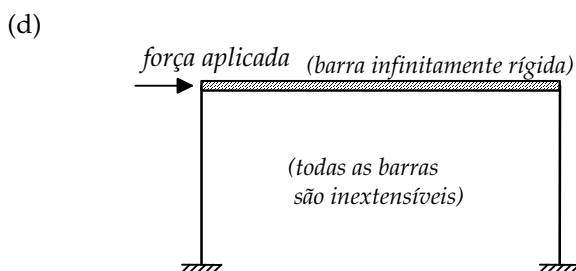
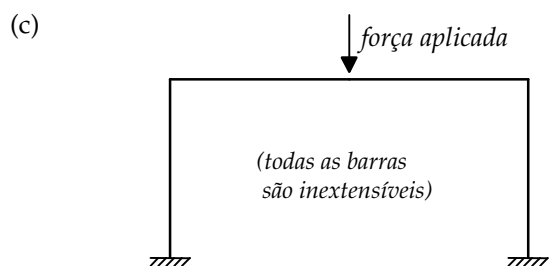
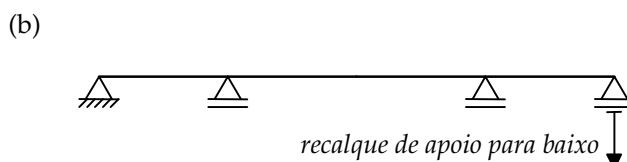
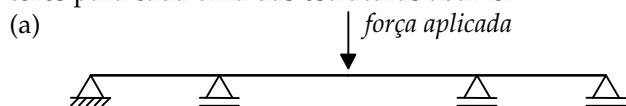
### 2ª Questão (2,0 pontos)

Empregando-se o Método das Forças, obter os diagramas de momentos fletores e momentos torçores para a grelha ao lado. Todas as barras têm a relação indicada entre a rigidez à torção  $GJ_t$  e a rigidez à flexão  $EI$ .



### 3ª Questão (1,5 ponto)

Desenhe os aspectos das configurações deformadas (de forma exagerada) e dos diagramas de momentos fletores para cada uma das estruturas abaixo.



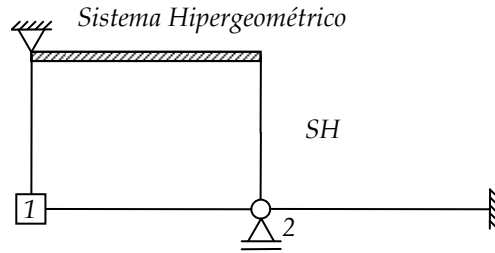
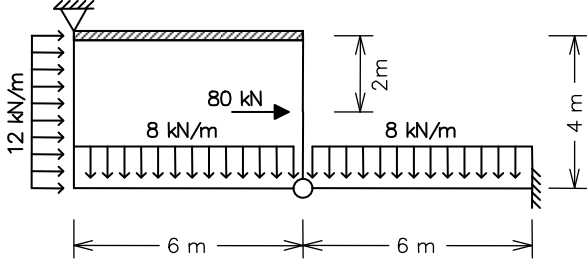
### 4ª Questão (1,0 ponto)

Grau vindo do segundo trabalho (nota do trabalho x 0,1).

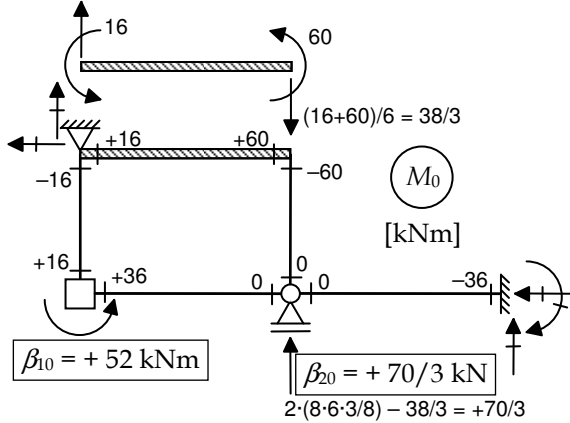
Solução de um sistema de 2 equações a 2 incógnitas:

$$\begin{Bmatrix} e \\ f \end{Bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} D_1 = \frac{bf - de}{ad - bc} \\ D_2 = \frac{ce - af}{ad - bc} \end{cases}$$

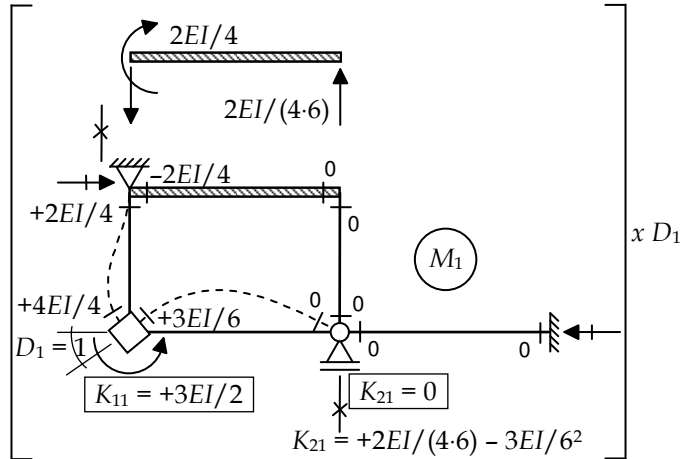
1ª Questão



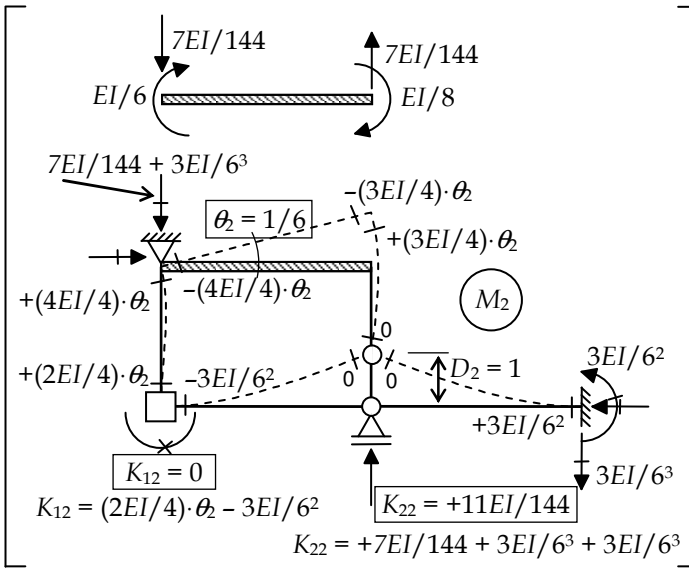
Caso (0) - Solicitação externa isolada no SH



Caso (1) - Deslocabilidade D1 isolada no SH



Caso (2) - Deslocabilidade D2 isolada no SH



Equações de equilíbrio:

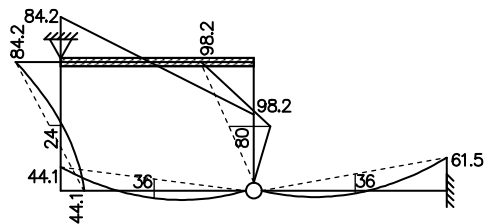
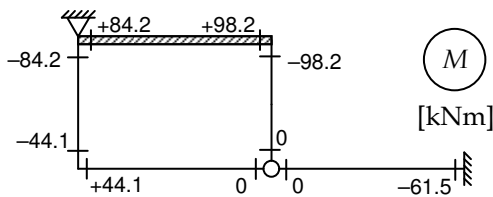
$$\begin{cases} \beta_{10} + K_{11}D_1 + K_{12}D_2 = 0 \\ \beta_{20} + K_{21}D_1 + K_{22}D_2 = 0 \end{cases}$$

$$\Rightarrow \begin{Bmatrix} +52 \\ +70 \\ +3 \end{Bmatrix} + EI \cdot \begin{bmatrix} +\frac{3}{2} & 0 \\ 0 & +\frac{11}{144} \end{bmatrix} \cdot \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

$$\Rightarrow \begin{cases} D_1 = -\frac{104}{3360} \\ D_2 = -\frac{3EI}{11EI} \end{cases}$$

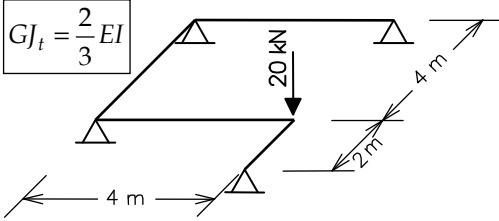
Momentos Fletores Finais:

$$M = M_0 + M_1 \cdot D_1 + M_2 \cdot D_2$$

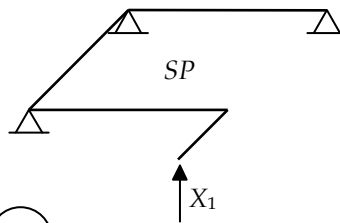


2ª Questão

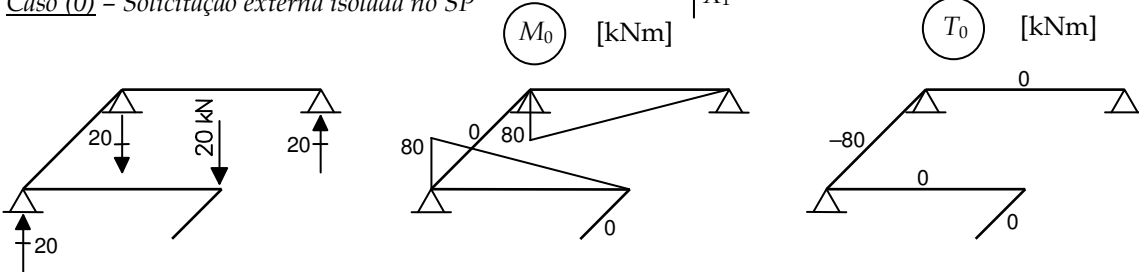
$$GJ_t = \frac{2}{3} EI$$



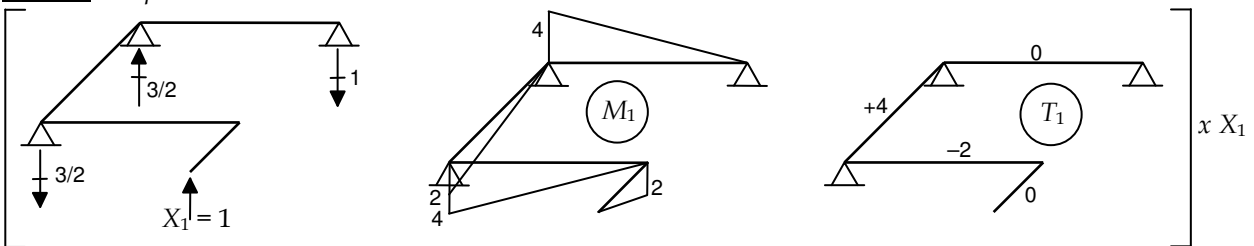
Sistema Principal e Hiperestático ( $g = 1$ )



Caso (0) - Solicitação externa isolada no SP



Caso (1) - Hiperestático  $X_1$  isolado no SP



Equação de compatibilidade:

$$\delta_{10} + \delta_{11} X_1 = 0$$

$$\delta_{10} = \left[ -\frac{1}{3} \cdot 4 \cdot 80 \cdot 4 - \frac{1}{3} \cdot 4 \cdot 80 \cdot 4 \right] \cdot \frac{1}{EI} + [(+4) \cdot (-80) \cdot 4] \cdot \frac{1}{GJ_t}$$

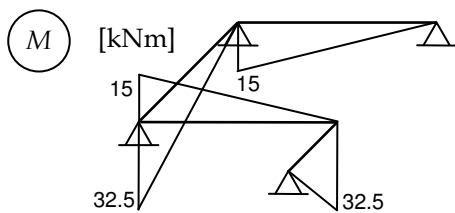
$$\delta_{10} = -\frac{2560}{3EI} - \frac{1280}{GJ_t} = -\frac{2560}{3EI} - \frac{3 \cdot 1280}{2EI} = -\frac{8320}{3EI}$$

$$\delta_{11} = \left[ 2 \cdot \left( +\frac{1}{3} \cdot 4 \cdot 4 \cdot 4 \right) + \frac{1}{3} \cdot 2 \cdot 2 \cdot 4 + \frac{1}{3} \cdot 2 \cdot 2 \cdot 2 \right] \cdot \frac{1}{EI} + [(+4) \cdot (+4) \cdot 4 + (-2) \cdot (-2) \cdot 4] \cdot \frac{1}{GJ_t}$$

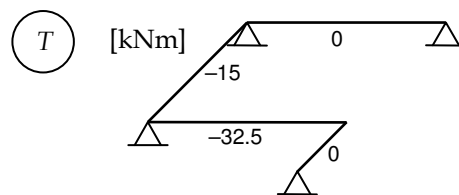
$$\delta_{11} = \frac{152}{3EI} + \frac{80}{GJ_t} = \frac{152}{3EI} + \frac{3 \cdot 80}{2EI} = +\frac{512}{3EI}$$

$$\Rightarrow -\frac{8320}{3EI} + \frac{512}{3EI} \cdot X_1 = 0 \quad \therefore X_1 = +16.25 \text{ kN}$$

Momentos Fletores Finais:  $M = M_0 + M_1 \cdot X_1$

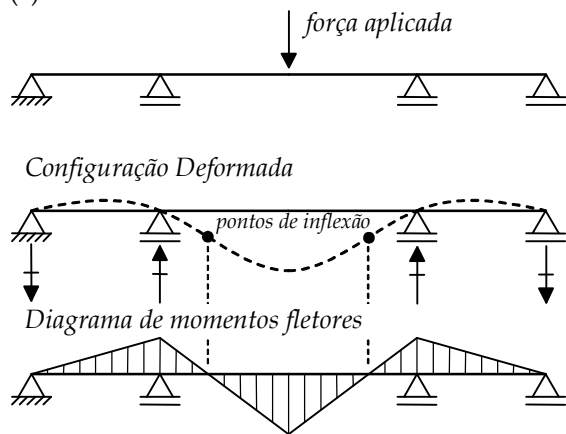


Momentos Torsores Finais:  $T = T_0 + T_1 \cdot X_1$

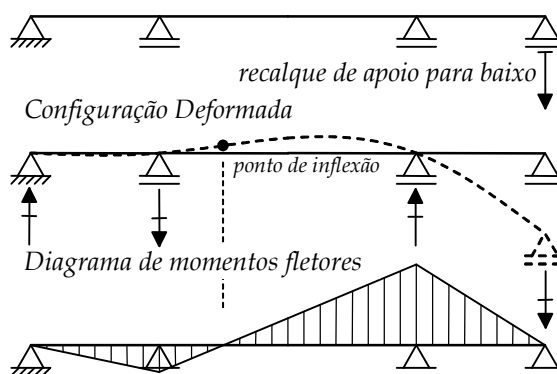


### 3ª Questão

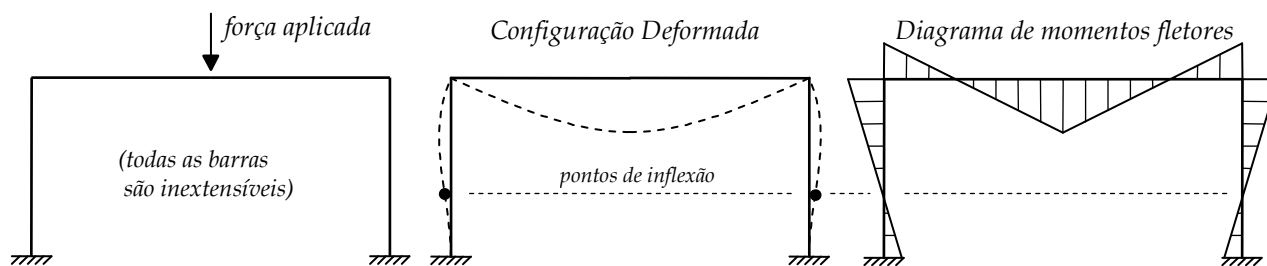
(a)



(b)



(c)



(d)

