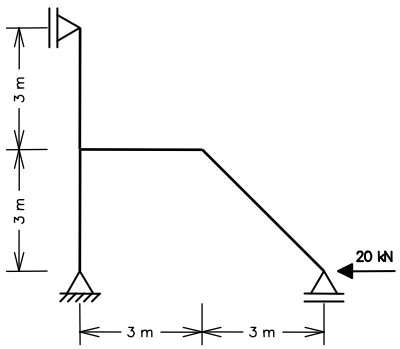
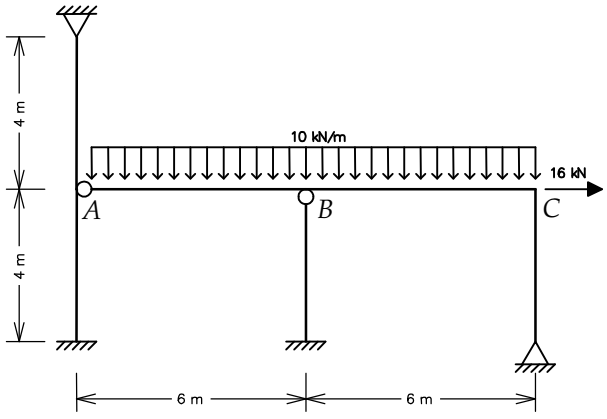
# CIV 1127 – ANÁLISE DE ESTRUTURAS II – 1º Semestre – 2008

## Prova Final – 02/07/2008 – Duração: 2:30 hs – Sem Consulta

**1ª Questão** (4,5 pontos)
Determine pelo Método das Forças o diagrama de momentos fletores do quadro hiperestático ao lado. Somente considere deformações por flexão. Todas as barras têm a mesma inércia à flexão $EI$ = 72000 kNm².



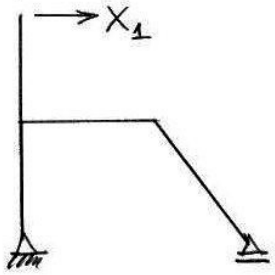

**2ª Questão** (5,5 pontos)
Empregando-se o Método dos Deslocamentos, obter o diagrama de momentos fletores para o quadro ao lado (barras inextensíveis). Todas as barras têm a mesma inércia à flexão $EI$ = 28800 kNm². Os valores das rotações e deslocamento horizontal dos nós $A$, $B$ e $C$ são fornecidos.



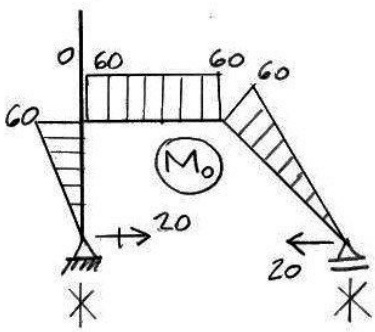

| Rotações | Deslocamento horizontal |
|---|---|
| $\theta_A = -4.703/EI = -1.633 \times 10^{-4}$ rad | $\Delta_A = +43.891/EI = +1.524 \times 10^{-3}$ m |
| $\theta_B = +9.078/EI = +3.152 \times 10^{-4}$ rad | $\Delta_B = +43.891/EI = +1.524 \times 10^{-3}$ m |
| $\theta_C = +13.231/EI = +4.594 \times 10^{-4}$ rad | $\Delta_C = +43.891/EI = +1.524 \times 10^{-3}$ m |

## 1ª Questão

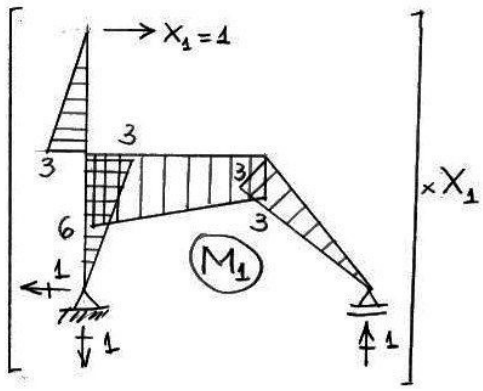
### Sistema Principal




### caso (0) - Solicitação externa isolada no S.P.




### Caso (1) - $X_1$ isolado no S.P.




### Eqn. de Compatibilidade

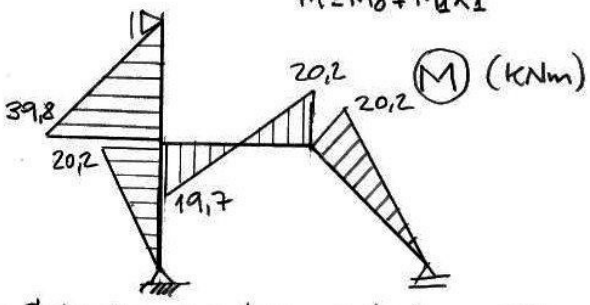
$$\delta_{10} + \delta_{11} X_1 = 0 \Rightarrow X_1 = \frac{990 + 180\sqrt{2}}{81 + 9\sqrt{2}} = 13{,}28$$

$$EI\delta_{10} = -\frac{1}{3} \times 3 \times 60 \times 3 - \left[1 \times 3 \times 60 \times 3 + \frac{1}{2} \times 3 \times 60 \times 3\right] - \frac{1}{3} \times 3\sqrt{2} \times 60 \times 3 = -(990 + 180\sqrt{2})$$

$$EI\delta_{11} = \frac{1}{3} \times 3 \times 3 \times 3 + \frac{1}{3} \times 3 \times 3 \times 3 + \left[1 \times 3 \times 3 \times 3 + 2 \times \frac{1}{2} \times 3 \times 3 \times 3 + \frac{1}{3} \times 3 \times 3 \times 3\right] + \frac{1}{3} \times 3\sqrt{2} \times 3 \times 3 = (81 + 9\sqrt{2})$$

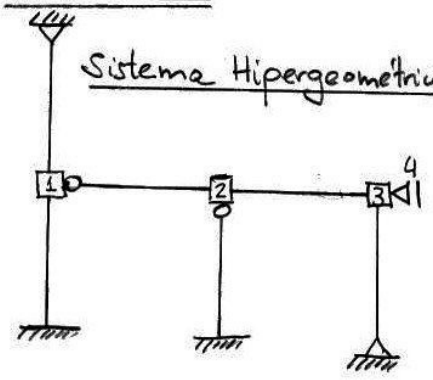
### Momentos Fletores Finais

$$M = M_0 + M_1 X_1$$

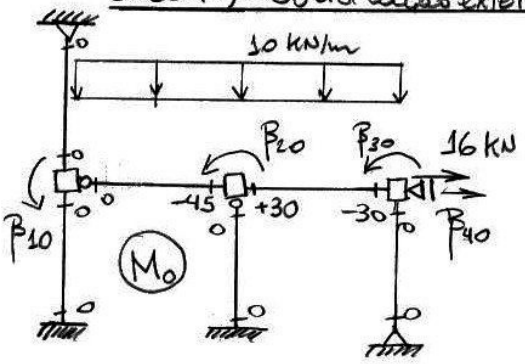



## 2ª Questão

### Sistema Hipergeométrico




### caso (0) - Solicitação externa isolada no S.H.



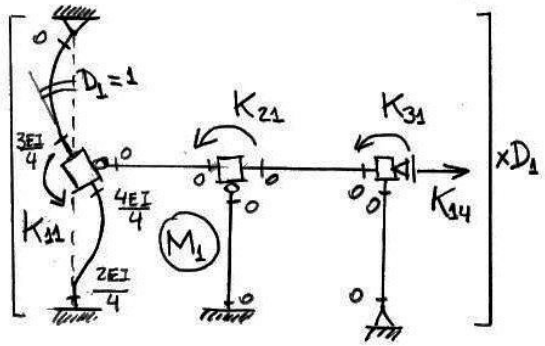

$\beta_{10} = 0$

$\beta_{20} = -45 + 30 = -15$ KNm

$\beta_{30} = -30$ KNm

$\beta_{40} = -16$ KN

### Caso (1) - $D_1$ isolada no S.H.

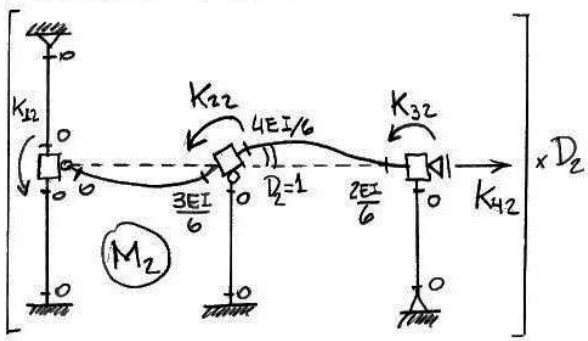



$$K_{11} = \frac{4EI}{4} + \frac{3EI}{4} = \frac{7EI}{4}$$

$$K_{21} = 0 \quad K_{31} = 0$$

$$K_{41} = \frac{6EI}{4^2} - \frac{3EI}{3^2} = \frac{3EI}{4^2} = \frac{3EI}{16}$$

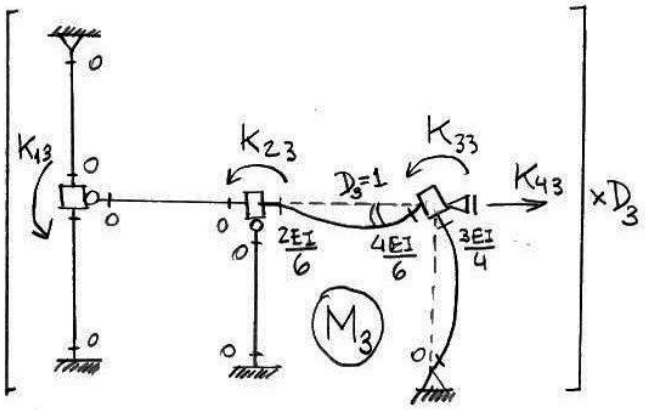
### caso (2) - $D_2$ isolada no S.H.




$$K_{12} = 0 \quad K_{22} = \frac{3EI}{6} + \frac{4EI}{6} = \frac{7EI}{6}$$

$$K_{32} = \frac{2EI}{6} \quad K_{42} = 0$$

caso (3) - $D_3$ isolada no S.H.

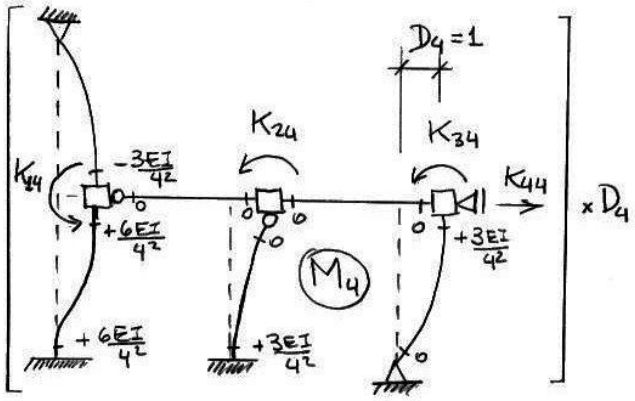



$K_{13} = 0 \qquad K_{23} = \frac{2EI}{6}$

$K_{33} = \frac{4EI}{6} + \frac{3EI}{4} = \frac{17EI}{12}$

$K_{43} = \frac{3EI}{4^2} = \frac{3EI}{16}$

caso (4) - $D_4$ isolada no S.H.




$K_{14} = \frac{6EI}{4^2} - \frac{3EI}{4^2} = \frac{3EI}{4^2} = \frac{3EI}{16}$

$K_{24} = 0 \qquad K_{34} = \frac{3EI}{4^2} = \frac{3EI}{16}$

$K_{44} = 3 \cdot \left[\frac{3EI}{4^3}\right] + \frac{12EI}{4^3} = \frac{21EI}{64}$

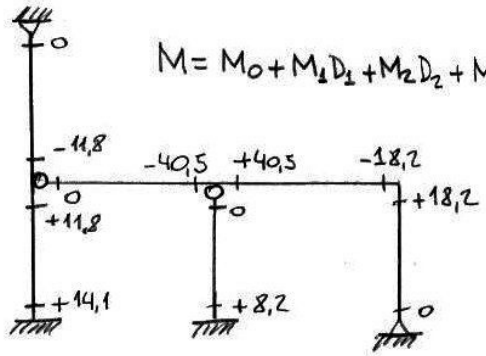
## Sistema de Eqns. de Equilíbrio

$$\begin{cases} \beta_{10} + K_{11}D_1 + K_{12}D_2 + K_{13}D_3 + K_{14}D_4 = 0 \\ \beta_{20} + K_{21}D_1 + K_{22}D_2 + K_{23}D_3 + K_{24}D_4 = 0 \\ \beta_{30} + K_{31}D_1 + K_{32}D_2 + K_{33}D_3 + K_{34}D_4 = 0 \\ \beta_{40} + K_{41}D_1 + K_{42}D_2 + K_{43}D_3 + K_{44}D_4 = 0 \end{cases}$$

$$\begin{Bmatrix} 0 \\ -15 \\ -30 \\ -16 \end{Bmatrix} + \begin{bmatrix} 7EI/4 & 0 & 0 & 3EI/16 \\ 0 & 7EI/6 & 2EI/6 & 0 \\ 0 & 2EI/6 & 17EI/12 & 3EI/16 \\ 3EI/16 & 0 & 3EI/16 & 21EI/64 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

## Momentos Fletores Finais

$$M = M_0 + M_1D_1 + M_2D_2 + M_3D_3 + M_4D_4$$




(KNm)