CIV 2802 – Sistemas Gráficos para Engenharia – 2008.1

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Condutividade Térmica 2D
Método das Diferenças Finitas – Técnica de Direções Alternadas

Parâmetros do modelo

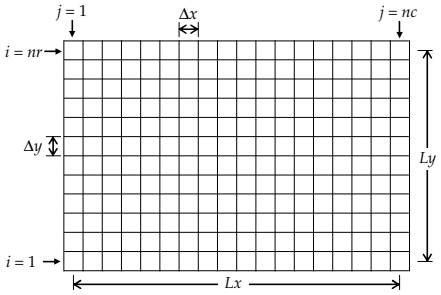
$u \rightarrow$ temperatura [°C]
$k \rightarrow$ condutividade térmica (considerado meio homegêneo) [W/(m °C)]
$\rho \rightarrow$ densidade do material (massa específica) [kg/m^3]
$C_P \rightarrow$ calor específico [J/kg °C]
$\alpha = \dfrac{k}{\rho \cdot C_P} \rightarrow$ difusividade térmica [m^2/s]
$h \rightarrow$ coeficiente de transferência de calor com o meio externo [W/(m^2 °C)]
$e \rightarrow$ espessura do meio bidimensional [m]
$Lx \rightarrow$ dimensão horizontal da placa (meio bidimensional) [m]
$Ly \rightarrow$ dimensão vertical da placa (meio bidimensional) [m]

Lei de Fourier

(fluxo térmico é taxa de energia por unidade de área)
$qx = -k \cdot \partial u/\partial x \rightarrow$ fluxo térmico na direção x [W/m^2]
$qy = -k \cdot \partial u/\partial y \rightarrow$ fluxo térmico na direção y [W/m^2]

Parâmetros da malha

$nr \rightarrow$ número de linhas (*rows*) do *grid*
$nc \rightarrow$ número de colunas do *grid*
$\Delta x = Lx/(nc-1) \rightarrow$ tamanho do lado da célula na direção x [m]
$\Delta y = Ly/(nr-1) \rightarrow$ tamanho do lado da célula na direção y [m]



Condições de contorno

$u(x=0,y) = ul \rightarrow$ temperatura constante prescrita no bordo esquerdo; ou
$qx(x=0,y) = ql \rightarrow$ fluxo de calor radiativo constante prescrito no bordo esquerdo; ou
$qx(x=0,y) = hl \cdot [u(x=0,y) - ul] \rightarrow$ troca de calor por convecção no bordo esquerdo

$u(x=Lx,y) = ur \rightarrow$ temperatura constante prescrita no bordo direito; ou
$qx(x=Lx,y) = qr \rightarrow$ fluxo de calor radiativo constante no bordo direito; ou
$qx(x=Lx,y) = hr \cdot [u(x=Lx,y) - ur] \rightarrow$ troca de calor por convecção no bordo direito

$u(x,y=0) = ub \rightarrow$ temperatura constante prescrita no bordo inferior; ou
$qy(x,y=0) = qb \rightarrow$ fluxo de calor radiativo constante no bordo inferior; ou
$qy(x,y=0) = hb \cdot [u(x,y=0) - ub] \rightarrow$ troca de calor por convecção no bordo inferior

$u(x,y=Ly) = ut \rightarrow$ temperatura constante prescrita no bordo superior; ou
$qy(x,y=Ly) = qt \rightarrow$ fluxo de calor radiativo constante no bordo superior; ou
$qy(x,y=Ly) = ht \cdot [u(x,y=Ly) - ut] \rightarrow$ troca de calor por convecção no bordo superior

Fonte de calor

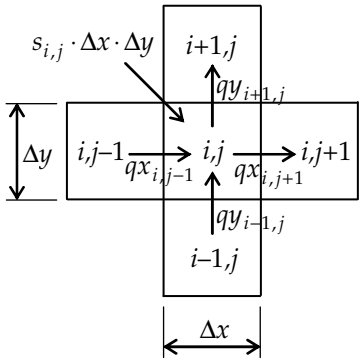
$s \rightarrow$ fonte de calor uniformemente distribuída por área [W/m^2]

Balanço de energia térmica em uma célula da malha:

(a célula é um volume de controle; convenção: calor que entra na célula é positivo)

[*Taxa com que energia entra na célula*] + [*Taxa de geração de energia dentro da célula (considerada nula)*] =
[*Taxa com que energia sai da célula*] + [*Taxa de armazenamento de energia dentro da célula*]

$$\left[qx_{i,j-1}\cdot\Delta y\cdot e+qy_{i-1,j}\cdot\Delta x\cdot e+s_{i,j}\cdot\Delta x\cdot\Delta y\right]+[0]=$$
$$\left[qx_{i,j+1}\cdot\Delta y\cdot e+qy_{i+1,j}\cdot\Delta x\cdot e\right]+\left[\rho\cdot C_P\cdot\Delta x\cdot\Delta y\cdot e\cdot\frac{\partial u}{\partial t}\right]$$



$$qx_{i,j-1}\cdot\Delta y\cdot e-qx_{i,j+1}\cdot\Delta y\cdot e+qy_{i-1,j}\cdot\Delta x\cdot e-qy_{i+1,j}\cdot\Delta x\cdot e=$$
$$\rho\cdot C_P\cdot\Delta x\cdot\Delta y\cdot e\cdot\frac{\partial u}{\partial t}-s_{i,j}\cdot\Delta x\cdot\Delta y$$

Usando a Lei de Fourier e a aproximação de derivadas em diferenças finitas:
$\left(qx\cong -k\cdot\Delta u/\Delta x\,;\quad qy\cong -k\cdot\Delta u/\Delta y\right)$

$$\left(-k\cdot\Delta y\cdot e\cdot\frac{u_{i,j}-u_{i,j-1}}{\Delta x}\right)-\left(-k\cdot\Delta y\cdot e\cdot\frac{u_{i,j+1}-u_{i,j}}{\Delta x}\right)+\left(-k\cdot\Delta x\cdot e\cdot\frac{u_{i,j}-u_{i-1,j}}{\Delta y}\right)-\left(-k\cdot\Delta x\cdot e\cdot\frac{u_{i+1,j}-u_{i,j}}{\Delta y}\right)=$$
$$\rho\cdot C_P\cdot\frac{\partial u}{\partial t}\cdot\Delta x\cdot\Delta y\cdot e-s_{i,j}\cdot\Delta x\cdot\Delta y$$

$$\left[\div\left(k\cdot\Delta x\cdot\Delta y\cdot e\right)\right]\;\Rightarrow\quad\frac{u_{i,j-1}-2u_{i,j}+u_{i,j+1}}{\Delta x^2}+\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{\Delta y^2}=\frac{\rho\cdot C_P}{k}\cdot\frac{\partial u}{\partial t}-\frac{s_{i,j}}{k\cdot e}$$

$$\Rightarrow\quad\boxed{\frac{u_{i,j-1}-2u_{i,j}+u_{i,j+1}}{\Delta x^2}+\frac{u_{i-1,j}-2u_{i,j}+u_{i+1,j}}{\Delta y^2}=\frac{1}{\alpha}\cdot\frac{\partial u}{\partial t}-\frac{s_{i,j}}{k\cdot e}}$$

Solução implícita da resposta transiente:

Considerando que os valores de temperatura em todos os pontos são conhecidos em um passo de tempo genérico (m), a aproximação da solução para o passo seguinte de tempo (m+1) resulta em um sistema de equações para solução no passo m+1, cuja matriz de coeficientes tem um formato "penta-diagonal" simétrico:

$$\begin{bmatrix}
X & X & X & & & & & & & & & & & \\
X & X & X & X & & & & & & & & & & \\
X & X & X & X & X & & & & & & & & & \\
 & X & X & X & X & X & & & & & & & & \\
 & & X & X & X & X & X & & & & & & & \\
 & & & X & X & X & X & X & & & & & & \\
 & & & & X & X & X & X & X & & & & & \\
 & & & & & X & X & X & X & X & & & & \\
 & & & & & & X & X & X & X & X & & & \\
 & & & & & & & X & X & X & X & X & & \\
 & & & & & & & & X & X & X & X & X & \\
 & & & & & & & & & X & X & X & X & \\
 & & & & & & & & & & X & X & X &
\end{bmatrix}\cdot\begin{Bmatrix}
\cdot\\ \cdot\\ \cdot\\ \cdot\\ \cdot\\ u_{i-1,j}^{m+1}\\ u_{i,j-1}^{m+1}\\ u_{i,j}^{m+1}\\ u_{i,j+1}^{m+1}\\ u_{i-1,j}^{m+1}\\ \cdot\\ \cdot\\ \cdot\\ \cdot
\end{Bmatrix}=\begin{Bmatrix}
X\\ X\\ X\\ X\\ X\\ X\\ X\\ X\\ X\\ X\\ X\\ X\\ X\\ X
\end{Bmatrix}$$

Esse sistema pode ser resolvido por métodos diretos (eliminação de Gauss, método de Cholesky, etc.) ou por métodos iterativos. Essa solução pode ser lenta do ponto de vista computacional se a malha for muito grande. Uma alternativa é a Técnica de Direções Alternadas que vai ser mostrada a seguir.

Técnica de Direções Alternadas

Ref.: Frind, E.O., *Groundwater Modelling (Numerical Methods)*, Lecture Notes Earth 456/656, Department of Earth Sciences, University of Waterloo, 1995.

1 – Resolvendo em x: (primeiro sub-passo no tempo m+1)

$$\underbrace{\frac{u_{i,j-1}^{m+1}-2u_{i,j}^{m+1}+u_{i,j+1}^{m+1}}{\Delta x^2}}_{\uparrow}+\underbrace{\frac{u_{i-1,j}^{m}-2u_{i,j}^{m}+u_{i+1}^{m}}{\Delta y^2}}_{\uparrow}=\frac{1}{\alpha}\cdot\underbrace{\frac{u_{i,j}^{m+1}}{\Delta t}}_{\uparrow}-\frac{1}{\alpha}\cdot\frac{u_{i,j}^{m}}{\Delta t}-\frac{s_{i,j}^{m}}{k\cdot e}$$

desconhecido — considerado igual ao do passo anterior — desconhecido

$$\Rightarrow\quad u_{i,j-1}^{m+1}-2u_{i,j}^{m+1}+u_{i,j+1}^{m+1}-\frac{1}{\alpha}\cdot\frac{u_{i,j}^{m+1}}{\Delta t}\cdot\Delta x^2=-\frac{\left(u_{i-1,j}^{m}-2u_{i,j}^{m}+u_{i+1,j}^{m}\right)}{\Delta y^2}\cdot\Delta x^2-\frac{1}{\alpha}\cdot\frac{u_{i,j}^{m}}{\Delta t}\cdot\Delta x^2-\frac{s_{i,j}^{m}}{k\cdot e}\cdot\Delta x^2$$

1.1 – Solução da linha genérica (i = 2:nr–1)

1.1.1 – Caso geral (células do interior da linha)

Adotando $\boxed{r_x=\frac{\alpha\cdot\Delta t}{\Delta x^2}}$, $\boxed{f_x=\frac{\Delta x^2}{\Delta y^2}}$ e $[\times(-1)]$ $\Rightarrow$

$$\boxed{(i=2:nr-1)\,(j=2:nc-1)}\rightarrow\boxed{-u_{i,j-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{i,j}^{m+1}-u_{i,j+1}^{m+1}=f_x\cdot u_{i-1,j}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{i,j}^{m}+f_x\cdot u_{i+1,j}^{m}+\frac{s_{i,j}^{m}}{k\cdot e}\cdot\Delta x^2}$$

1.1.2 – Condições de contorno no bordo esquerdo

1.1.2.1 – Condição de contorno de temperatura constante prescrita no bordo esquerdo

$$\boxed{(i=2:nr-1)\quad(j=1)}\;\rightarrow\;\boxed{u_{i,1}=ul}$$

1.1.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo esquerdo

$$qx=-k\cdot\frac{du}{dx}\cdot n_x\;\rightarrow\;qx_{i,1}\cong-k\cdot\frac{u_{i,2}-u_{i,0}}{2\Delta x}\cdot(-1)=ql\;\rightarrow\;u_{i,0}=u_{i,2}-ql\cdot\frac{2\Delta x}{k}$$

$$-u_{i,0}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{i,1}^{m+1}-u_{i,2}^{m+1}=f_x\cdot u_{i-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{i,1}^{m}+f_x\cdot u_{i+1,1}^{m}+\frac{s_{i,1}^{m}}{k\cdot e}\cdot\Delta x^2\quad\Rightarrow$$

$$-\left(u_{i,2}^{m+1}-ql\cdot\frac{2\Delta x}{k}\right)+\left(2+\frac{1}{r_x}\right)\cdot u_{i,1}^{m+1}-u_{i,2}^{m+1}=f_x\cdot u_{i-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{i,1}^{m}+f_x\cdot u_{i+1,1}^{m}+\frac{s_{i,1}^{m}}{k\cdot e}\cdot\Delta x^2\quad\Rightarrow$$

$$\boxed{(i=2:nr-1)\quad(j=1)}\;\rightarrow\;\boxed{\left(2+\frac{1}{r_x}\right)\cdot u_{i,1}^{m+1}-2u_{i,2}^{m+1}=f_x\cdot u_{i-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{i,1}^{m}+f_x\cdot u_{i+1,1}^{m}+\frac{s_{i,1}^{m}}{k\cdot e}\cdot\Delta x^2-ql\cdot\frac{2\Delta x}{k}}$$

1.1.2.3 – Condição de contorno de troca de calor por convecção no bordo esquerdo

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hl\cdot(u-ul)\;\rightarrow\;-k\cdot\frac{u_{i,2}-u_{i,0}}{2\Delta x}\cdot(-1)=hl\cdot\left(u_{i,1}-ul\right)\;\rightarrow\;u_{i,0}=-\frac{2\Delta x\cdot hl}{k}\cdot u_{i,1}+u_{i,2}+\frac{2\Delta x\cdot hl}{k}\cdot ul\quad\Rightarrow$$

$$-\left(-\frac{2\Delta x\cdot hl}{k}\cdot u_{i,1}^{m+1}+u_{i,2}^{m+1}+\frac{2\Delta x\cdot hl}{k}\cdot ul\right)+\left(2+\frac{1}{r_x}\right)\cdot u_{i,1}^{m+1}-u_{i,2}^{m+1}=f_x\cdot u_{i-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{i,1}^{m}+f_x\cdot u_{i+1,1}^{m}+\frac{s_{i,1}^{m}}{k\cdot e}\cdot\Delta x^2\quad\Rightarrow$$

$$\boxed{(i=2:nr-1)\quad(j=1)}\;\rightarrow\;\boxed{\left(2+\frac{1}{r_x}+\frac{2\Delta x\cdot hl}{k}\right)\cdot u_{i,1}^{m+1}-2u_{i,2}^{m+1}=f_x\cdot u_{i-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{i,1}^{m}+f_x\cdot u_{i+1,1}^{m}+\frac{s_{i,1}^{m}}{k\cdot e}\cdot\Delta x^2+\frac{2\Delta x\cdot hl}{k}\cdot ul}$$

1.1.3 – Condições de contorno no bordo direito

1.1.3.1 – Condição de contorno de temperatura constante prescrita no bordo direito

$$\boxed{(i=2:nr-1)\quad(j=nc)}\;\rightarrow\;\boxed{u_{i,nc}=ur}$$

1.1.3.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo direito

$$qx = -k \cdot \frac{du}{dx} \cdot n_x \;\rightarrow\; qx_{i,nc} \cong -k \cdot \frac{u_{i,nc+1} - u_{i,nc-1}}{2\Delta x} \cdot (+1) = qr \;\rightarrow\; u_{i,nc+1} = u_{i,nc-1} - qr \cdot \frac{2\Delta x}{k}$$

$$-u_{i,nc-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{i,nc}^{m+1} - u_{i,nc+1}^{m+1} = f_x \cdot u_{i-1,nc}^{m} + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{i,nc}^{m} + f_x \cdot u_{i+1,nc}^{m} + \frac{s_{i,nc}^{m}}{k \cdot e} \cdot \Delta x^2 \;\Rightarrow$$

$$-u_{i,nc-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{i,nc}^{m+1} - \left(u_{i,nc-1}^{m+1} - qr \cdot \frac{2\Delta x}{k}\right) = f_x \cdot u_{i-1,nc}^{m} + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{i,nc}^{m} + f_x \cdot u_{i+1,nc}^{m} + \frac{s_{i,nc}^{m}}{k \cdot e} \cdot \Delta x^2 \;\Rightarrow$$

$$\boxed{(i = 2 : nr-1) \quad (j = nc)} \rightarrow \boxed{-2u_{i,nc-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{i,nc}^{m+1} = f_x \cdot u_{i-1,nc}^{m} + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{i,nc}^{m} + f_x \cdot u_{i+1,nc}^{m} + \frac{s_{i,nc}^{m}}{k \cdot e} \cdot \Delta x^2 - qr \cdot \frac{2\Delta x}{k}}$$

1.1.3.3 – Condição de contorno de troca de calor por convecção no bordo direito

$$qx = -k \cdot \frac{du}{dx} \cdot n_x = hr \cdot (u - ur) \;\rightarrow\; -k \cdot \frac{u_{i,nc+1} - u_{i,nc-1}}{2\Delta x} \cdot (+1) = hr \cdot (u_{i,nc} - ur) \;\rightarrow\; u_{i,nc+1} = u_{i,nc-1} - \frac{2\Delta x \cdot hr}{k} \cdot u_{i,nc} + \frac{2\Delta x \cdot hr}{k} \cdot ur \;\Rightarrow$$

$$-u_{i,nc-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{i,nc}^{m+1} - \left(u_{i,nc-1}^{m+1} - \frac{2\Delta x \cdot hr}{k} \cdot u_{i,nc}^{m+1} + \frac{2\Delta x \cdot hr}{k} \cdot ur\right) = f_x \cdot u_{i-1,nc}^{m} + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{i,nc}^{m} + f_x \cdot u_{i+1,nc}^{m} + \frac{s_{i,nc}^{m}}{k \cdot e} \cdot \Delta x^2 \;\Rightarrow$$

$$\boxed{(i = 2 : nr-1) \quad (j = nc)} \rightarrow \boxed{-2u_{i,nc-1}^{m+1} + \left(2 + \frac{1}{r_x} + \frac{2\Delta x \cdot hr}{k}\right) \cdot u_{i,nc}^{m+1} = f_x \cdot u_{i-1,nc}^{m} + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{i,nc}^{m} + f_x \cdot u_{i+1,nc}^{m} + \frac{s_{i,nc}^{m}}{k \cdot e} \cdot \Delta x^2 + \frac{2\Delta x \cdot hr}{k} \cdot ur}$$

1.2 – Solução da linha inferior ($i = 1$)

1.2.1 – Condição de contorno de temperatura constante prescrita no bordo inferior

$$\boxed{(i = 1) \quad (j = 1 : nc)} \;\rightarrow\; \boxed{u_{1,j} = ub}$$

1.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo inferior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y \;\rightarrow\; qy_{1,j} \cong -k \cdot \frac{u_{2,j} - u_{0,j}}{2\Delta y} \cdot (-1) = qb \;\rightarrow\; u_{0,j} = u_{2,j} - qb \cdot \frac{2\Delta y}{k}$$

$$-u_{1,j-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,j}^{m+1} - u_{1,j+1}^{m+1} = f_x \cdot u_{0,j}^{m} + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,j}^{m} + f_x \cdot u_{2,j}^{m} + \frac{s_{1,j}^{m}}{k \cdot e} \cdot \Delta x^2 \;\Rightarrow$$

$$-u_{1,j-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,j}^{m+1} - u_{1,j+1}^{m+1} = f_x \cdot \left(u_{2,j}^{m} - qb \cdot \frac{2\Delta y}{k}\right) + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,j}^{m} + f_x \cdot u_{2,j}^{m} + \frac{s_{1,j}^{m}}{k \cdot e} \cdot \Delta x^2 \;\Rightarrow$$

$$\boxed{(i = 1) \quad (j = 2 : nc-1)} \;\rightarrow\; \boxed{-u_{1,j-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,j}^{m+1} - u_{1,j+1}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,j}^{m} + 2f_x \cdot u_{2,j}^{m} + \frac{s_{1,j}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k}}$$

1.2.2.1 – Condições de contorno no bordo esquerdo

1.2.2.1.1 – Condição de contorno de temperatura constante prescrita no bordo esquerdo

$$\boxed{(i = 1) \quad (j = 1)} \;\rightarrow\; \boxed{u_{1,1} = ul}$$

1.2.2.1.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo esquerdo

$$qx = -k \cdot \frac{du}{dx} \cdot n_x \;\rightarrow\; qx_{1,1} \cong -k \cdot \frac{u_{1,2} - u_{1,0}}{2\Delta x} \cdot (-1) = ql \;\rightarrow\; u_{1,0} = u_{1,2} - ql \cdot \frac{2\Delta x}{k}$$

$$-u_{1,0}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,1}^{m+1} - u_{1,2}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,1}^{m} + 2f_x \cdot u_{2,1}^{m} + \frac{s_{1,1}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} \;\Rightarrow$$

$$-\left(u_{1,2}^{m+1} - ql \cdot \frac{2\Delta x}{k}\right) + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,1}^{m+1} - u_{1,2}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,1}^{m} + 2f_x \cdot u_{2,1}^{m} + \frac{s_{1,1}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} \;\Rightarrow$$

$$\boxed{(i = 1) \quad (j = 1)} \;\rightarrow\; \boxed{\left(2 + \frac{1}{r_x}\right) \cdot u_{1,1}^{m+1} - 2u_{1,2}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,1}^{m} + 2f_x \cdot u_{2,1}^{m} + \frac{s_{1,1}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} - ql \cdot \frac{2\Delta x}{k}}$$

1.2.2.1.3 – Condição de contorno de troca de calor por convecção no bordo esquerdo

$$qx = -k \cdot \frac{du}{dx} \cdot n_x = hl \cdot (u - ul) \;\rightarrow\; -k \cdot \frac{u_{1,2} - u_{1,0}}{2\Delta x} \cdot (-1) = hl \cdot (u_{1,1} - ul) \;\rightarrow\; u_{1,0} = -\frac{2\Delta x \cdot hl}{k} \cdot u_{1,1} + u_{1,2} + \frac{2\Delta x \cdot hl}{k} \cdot ul \;\Rightarrow$$

$$-\left(-\frac{2\Delta x \cdot hl}{k} \cdot u_{1,1}^{m+1} + u_{1,2}^{m+1} + \frac{2\Delta x \cdot hl}{k} \cdot ul\right) + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,1}^{m+1} - u_{1,2}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,1}^{m} + 2f_x \cdot u_{2,1}^{m} + \frac{s_{1,1}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} \;\Rightarrow$$

$$\boxed{(i=1) \quad (j=1)} \;\rightarrow\; \boxed{\left(2 + \frac{1}{r_x} + \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{1,1}^{m+1} - 2u_{1,2}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,1}^{m} + 2f_x \cdot u_{2,1}^{m} + \frac{s_{1,1}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} + \frac{2\Delta x \cdot hl}{k} \cdot ul}$$

1.2.2.2 – Condições de contorno no bordo direito

1.2.2.2.1 – Condição de contorno de temperatura constante prescrita no bordo direito

$$\boxed{(i=1) \quad (j=nc)} \;\rightarrow\; \boxed{u_{1,nc} = ur}$$

1.2.2.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo direito

$$qx = -k \cdot \frac{du}{dx} \cdot n_x \;\rightarrow\; qx_{1,nc} \cong -k \cdot \frac{u_{1,nc+1} - u_{1,nc-1}}{2\Delta x} \cdot (+1) = qr \;\rightarrow\; u_{1,nc+1} = u_{1,nc-1} - qr \cdot \frac{2\Delta x}{k}$$

$$-u_{1,nc-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,nc}^{m+1} - u_{1,nc+1}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,nc}^{m} + 2f_x \cdot u_{2,nc}^{m} + \frac{s_{1,nc}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} \;\Rightarrow$$

$$-u_{1,nc-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,nc}^{m+1} - \left(u_{1,nc-1}^{m+1} - qr \cdot \frac{2\Delta x}{k}\right) = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,nc}^{m} + 2f_x \cdot u_{2,nc}^{m} + \frac{s_{1,nc}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} \;\Rightarrow$$

$$\boxed{(i=1) \quad (j=nc)} \;\rightarrow\; \boxed{-2u_{1,nc-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,nc}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,nc}^{m} + 2f_x \cdot u_{2,nc}^{m} + \frac{s_{1,nc}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} - qr \cdot \frac{2\Delta x}{k}}$$

1.2.2.2.3 – Condição de contorno de troca de calor por convecção no bordo direito

$$qx = -k \cdot \frac{du}{dx} \cdot n_x = hr \cdot (u - ur) \;\rightarrow\; -k \cdot \frac{u_{1,nc+1} - u_{1,nc-1}}{2\Delta x} \cdot (+1) = hr \cdot (u_{1,nc} - ur) \;\rightarrow\; u_{1,nc+1} = u_{1,nc-1} - \frac{2\Delta x \cdot hr}{k} \cdot u_{1,nc} + \frac{2\Delta x \cdot hr}{k} \cdot ur \;\Rightarrow$$

$$-u_{1,nc-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,nc}^{m+1} - \left(u_{1,nc-1}^{m+1} - \frac{2\Delta x \cdot hr}{k} \cdot u_{1,nc}^{m+1} + \frac{2\Delta x \cdot hr}{k} \cdot ur\right) = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,nc}^{m} + 2f_x \cdot u_{2,nc}^{m} + \frac{s_{1,nc}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} \;\Rightarrow$$

$$\boxed{(i=1) \quad (j=nc)} \;\rightarrow\; \boxed{-2u_{1,nc-1}^{m+1} + \left(2 + \frac{1}{r_x} + \frac{2\Delta x \cdot hr}{k}\right) \cdot u_{1,nc}^{m+1} = \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,nc}^{m} + 2f_x \cdot u_{2,nc}^{m} + \frac{s_{1,nc}^{m}}{k \cdot e} \cdot \Delta x^2 - qb \cdot f_x \cdot \frac{2\Delta y}{k} + \frac{2\Delta x \cdot hr}{k} \cdot ur}$$

1.2.3 – Condição de contorno de troca de calor por convecção no bordo inferior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y = hb \cdot (u - ub) \rightarrow\; -k \cdot \frac{u_{2,j} - u_{0,j}}{2\Delta y} \cdot (-1) = hb \cdot (u_{1,j} - ub) \;\rightarrow\; u_{0,j} = -\frac{2\Delta y \cdot hb}{k} \cdot u_{1,j} + u_{2,j} + \frac{2\Delta y \cdot hb}{k} \cdot ub$$

$$-u_{1,j-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,j}^{m+1} - u_{1,j+1}^{m+1} = f_x \cdot u_{0,j}^{m} + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,j}^{m} + f_x \cdot u_{2,j}^{m} + \frac{s_{1,j}^{m}}{k \cdot e} \cdot \Delta x^2 \;\Rightarrow$$

$$-u_{1,j-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,j}^{m+1} - u_{1,j+1}^{m+1} = f_x \cdot \left(-\frac{2\Delta y \cdot hb}{k} \cdot u_{1,j}^{m} + u_{2,j}^{m} + \frac{2\Delta y \cdot hb}{k} \cdot ub\right) + \left(\frac{1}{r_x} - 2f_x\right) \cdot u_{1,j}^{m} + f_x \cdot u_{2,j}^{m} + \frac{s_{1,j}^{m}}{k \cdot e} \cdot \Delta x^2 \;\Rightarrow$$

$$\boxed{(i=1) \quad (j=2:nc-1)} \;\rightarrow\; \boxed{-u_{1,j-1}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,j}^{m+1} - u_{1,j+1}^{m+1} = \left(\frac{1}{r_x} - 2f_x - f_x \cdot \frac{2\Delta y \cdot hb}{k}\right) \cdot u_{1,j}^{m} + 2f_x \cdot u_{2,j}^{m} + \frac{s_{1,j}^{m}}{k \cdot e} \cdot \Delta x^2 + f_x \cdot \frac{2\Delta y \cdot hb}{k} \cdot ub}$$

1.2.3.1 – Condições de contorno no bordo esquerdo

1.2.3.1.1 – Condição de contorno de temperatura constante prescrita no bordo esquerdo

$$\boxed{(i=1) \quad (j=1)} \;\rightarrow\; \boxed{u_{1,1} = ul}$$

1.2.3.1.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo esquerdo

$$qx = -k \cdot \frac{du}{dx} \cdot n_x \;\rightarrow\; qx_{1,1} \cong -k \cdot \frac{u_{1,2} - u_{1,0}}{2\Delta x} \cdot (-1) = ql \;\rightarrow\; u_{1,0} = u_{1,2} - ql \cdot \frac{2\Delta x}{k}$$

$$-u_{1,0}^{m+1} + \left(2 + \frac{1}{r_x}\right) \cdot u_{1,1}^{m+1} - u_{1,2}^{m+1} = \left(\frac{1}{r_x} - 2f_x - f_x \cdot \frac{2\Delta y \cdot hb}{k}\right) \cdot u_{1,1}^{m} + 2f_x \cdot u_{2,1}^{m} + \frac{s_{1,1}^{m}}{k \cdot e} \cdot \Delta x^2 + f_x \cdot \frac{2\Delta y \cdot hb}{k} \cdot ub \;\Rightarrow$$

$$-\left(u_{1,2}^{m+1}-ql\cdot\frac{2\Delta x}{k}\right)+\left(2+\frac{1}{r_x}\right)\cdot u_{1,1}^{m+1}-u_{1,2}^{m+1}=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,1}^{m}+2f_x\cdot u_{2,1}^{m}+\frac{s_{1,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub \;\Rightarrow$$

$$\boxed{(i=1)\quad(j=1)}\;\rightarrow\;\boxed{\left(2+\frac{1}{r_x}\right)\cdot u_{1,1}^{m+1}-2u_{1,2}^{m+1}=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,1}^{m}+2f_x\cdot u_{2,1}^{m}+\frac{s_{1,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub-ql\cdot\frac{2\Delta x}{k}}$$

1.2.3.1.3 – Condição de contorno de troca de calor por convecção no bordo esquerdo

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hl\cdot(u-ul)\;\rightarrow\;-k\cdot\frac{u_{1,2}-u_{1,0}}{2\Delta x}\cdot(-1)=hl\cdot(u_{1,1}-ul)\;\rightarrow\;u_{1,0}=-\frac{2\Delta x\cdot hl}{k}\cdot u_{1,1}+u_{1,2}+\frac{2\Delta x\cdot hl}{k}\cdot ul\;\Rightarrow$$

$$-\left(-\frac{2\Delta x\cdot hl}{k}\cdot u_{1,1}^{m+1}+u_{1,2}^{m+1}+\frac{2\Delta x\cdot hl}{k}\cdot ul\right)+\left(2+\frac{1}{r_x}\right)\cdot u_{1,1}^{m+1}-u_{1,2}^{m+1}=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,1}^{m}+2f_x\cdot u_{2,1}^{m}+\frac{s_{1,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub\;\Rightarrow$$

$$\boxed{(i=1)\quad(j=1)}\;\rightarrow\;\boxed{\left(2+\frac{1}{r_x}+\frac{2\Delta x\cdot hl}{k}\right)\cdot u_{1,1}^{m+1}-2u_{1,2}^{m+1}=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,1}^{m}+2f_x\cdot u_{2,1}^{m}+\frac{s_{1,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub+\frac{2\Delta x\cdot hl}{k}\cdot ul}$$

1.2.3.2 – Condições de contorno no bordo direito

1.2.3.2.1 – Condição de contorno de temperatura constante prescrita no bordo direito

$$\boxed{(i=1)\quad(j=nc)}\;\rightarrow\;\boxed{u_{1,nc}=ur}$$

1.2.3.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo direito

$$qx=-k\cdot\frac{du}{dx}\cdot n_x\;\rightarrow\;qx_{1,nc}\cong-k\cdot\frac{u_{1,nc+1}-u_{1,nc-1}}{2\Delta x}\cdot(+1)=qr\;\rightarrow\;u_{1,nc+1}=u_{1,nc-1}-qr\cdot\frac{2\Delta x}{k}$$

$$-u_{1,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{1,nc}^{m+1}-u_{1,nc+1}^{m+1}=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,nc}^{m}+2f_x\cdot u_{2,nc}^{m}+\frac{s_{1,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub\;\Rightarrow$$

$$-u_{1,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{1,nc}^{m+1}-\left(u_{1,nc-1}^{m+1}-qr\cdot\frac{2\Delta x}{k}\right)=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,nc}^{m}+2f_x\cdot u_{2,nc}^{m}+\frac{s_{1,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub\;\Rightarrow$$

$$\boxed{(i=1)\quad(j=nc)}\;\rightarrow\;\boxed{-2u_{1,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{1,nc}^{m+1}=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,nc}^{m}+2f_x\cdot u_{2,nc}^{m}+\frac{s_{1,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub-qr\cdot\frac{2\Delta x}{k}}$$

1.2.3.2.3 – Condição de contorno de troca de calor por convecção no bordo direito

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hr\cdot(u-ur)\;\rightarrow\;-k\cdot\frac{u_{1,nc+1}-u_{1,nc-1}}{2\Delta x}\cdot(+1)=hr\cdot(u_{1,nc}-ur)\;\rightarrow\;u_{1,nc+1}=u_{1,nc-1}-\frac{2\Delta x\cdot hr}{k}\cdot u_{1,nc}+\frac{2\Delta x\cdot hr}{k}\cdot ur\;\Rightarrow$$

$$-u_{1,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{1,nc}^{m+1}-\left(u_{1,nc-1}^{m+1}-\frac{2\Delta x\cdot hr}{k}\cdot u_{1,nc}^{m+1}+\frac{2\Delta x\cdot hr}{k}\cdot ur\right)=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,nc}^{m}+2f_x\cdot u_{2,nc}^{m}+\frac{s_{1,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub\;\Rightarrow$$

$$\boxed{(i=1)\quad(j=nc)}\;\rightarrow\;\boxed{-2u_{1,nc-1}^{m+1}+\left(2+\frac{1}{r_x}+\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{1,nc}^{m+1}=\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,nc}^{m}+2f_x\cdot u_{2,nc}^{m}+\frac{s_{1,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot hb}{k}\cdot ub+\frac{2\Delta x\cdot hr}{k}\cdot ur}$$

1.3 – Solução da linha superior ($i = nr$)

1.3.1 – Condição de contorno de temperatura constante prescrita no bordo superior

$$\boxed{(i=nr)\quad(j=1:nc)}\;\rightarrow\;\boxed{u_{nr,j}=ut}$$

1.3.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo superior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y\rightarrow qy_{nr,j}\cong-k\cdot\frac{u_{nr+1,j}-u_{nr-1,j}}{2\Delta y}\cdot(+1)=qt\;\rightarrow\;u_{nr+1,j}=u_{nr-1,j}-qt\cdot\frac{2\Delta y}{k}$$

$$-u_{nr,j-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,j}^{m+1}-u_{nr,j+1}^{m+1}=f_x\cdot u_{nr-1,j}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,j}^{m}+f_x\cdot u_{nr+1,j}^{m}+\frac{s_{nr,j}^{m}}{k\cdot e}\cdot\Delta x^2\;\Rightarrow$$

$$-u_{nr,j-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,j}^{m+1}-u_{nr,j+1}^{m+1}=f_x\cdot u_{nr-1,j}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,j}^{m}+f_x\cdot\left(u_{nr-1,j}^{m}-qt\cdot\frac{2\Delta y}{k}\right)+\frac{s_{nr,j}^{m}}{k\cdot e}\cdot\Delta x^2\;\Rightarrow$$

$$\boxed{(i=nr)\quad(j=2:nc-1)}\rightarrow\boxed{-u_{nr,j-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,j}^{m+1}-u_{nr,j+1}^{m+1}=2f_x\cdot u_{nr-1,j}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,j}^{m}+\frac{s_{nr,j}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}}$$

1.3.2.1 – Condições de contorno no bordo esquerdo

1.3.2.1.1 – Condição de contorno de temperatura constante prescrita no bordo esquerdo

$$\boxed{(i=nr)\quad(j=1)}\ \rightarrow\ \boxed{u_{nr,1}=ul}$$

1.3.2.1.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo esquerdo

$$qx=-k\cdot\frac{du}{dx}\cdot n_x\ \rightarrow\ qx_{nr,1}\cong -k\cdot\frac{u_{nr,2}-u_{nr,0}}{2\Delta x}\cdot(-1)=ql\ \rightarrow\ u_{nr,0}=u_{nr,2}-ql\cdot\frac{2\Delta x}{k}$$

$$-u_{nr,0}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,1}^{m+1}-u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}\ \Rightarrow$$

$$-\left(u_{nr,2}^{m+1}-ql\cdot\frac{2\Delta x}{k}\right)+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,1}^{m+1}-u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}\ \Rightarrow$$

$$\boxed{(i=nr)\quad(j=1)}\ \rightarrow\ \boxed{\left(2+\frac{1}{r_x}\right)\cdot u_{nr,1}^{m+1}-2u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}-ql\cdot\frac{2\Delta x}{k}}$$

1.3.2.1.3 – Condição de contorno de troca de calor por convecção no bordo esquerdo

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hl\cdot(u-ul)\ \rightarrow\ -k\cdot\frac{u_{nr,2}-u_{nr,0}}{2\Delta x}\cdot(-1)=hl\cdot\left(u_{nr,1}-ul\right)\ \rightarrow\ u_{nr,0}=-\frac{2\Delta x\cdot hl}{k}\cdot u_{nr,1}+u_{nr,2}+\frac{2\Delta x\cdot hl}{k}\cdot ul\ \Rightarrow$$

$$-\left(-\frac{2\Delta x\cdot hl}{k}\cdot u_{nr,1}^{m+1}+u_{nr,2}^{m+1}+\frac{2\Delta x\cdot hl}{k}\cdot ul\right)+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,1}^{m+1}-u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}\ \Rightarrow$$

$$\boxed{(i=nr)\quad(j=1)}\ \rightarrow\ \boxed{\left(2+\frac{1}{r_x}+\frac{2\Delta x\cdot hl}{k}\right)\cdot u_{nr,1}^{m+1}-2u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}+\frac{2\Delta x\cdot hl}{k}\cdot ul}$$

1.3.2.2 – Condições de contorno no bordo direito

1.3.2.2.1 – Condição de contorno de temperatura constante prescrita no bordo direito

$$\boxed{(i=nr)\quad(j=nc)}\ \rightarrow\ \boxed{u_{nr,nc}=ur}$$

1.3.2.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo direito

$$qx=-k\cdot\frac{du}{dx}\cdot n_x\ \rightarrow\ qx_{nr,nc}\cong -k\cdot\frac{u_{nr,nc+1}-u_{nr,nc-1}}{2\Delta x}\cdot(+1)=qr\ \rightarrow\ u_{nr,nc+1}=u_{nr,nc-1}-qr\cdot\frac{2\Delta x}{k}$$

$$-u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,nc}^{m+1}-u_{nr,nc+1}^{m+1}=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}\ \Rightarrow$$

$$-u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,nc}^{m+1}-\left(u_{nr,nc-1}^{m+1}-qr\cdot\frac{2\Delta x}{k}\right)=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}\ \Rightarrow$$

$$\boxed{(i=nr)\quad(j=nc)}\ \rightarrow\ \boxed{-2u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,nc}^{m+1}=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}-qr\cdot\frac{2\Delta x}{k}}$$

1.3.2.2.3 – Condição de contorno de troca de calor por convecção no bordo direito

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hr\cdot(u-ur)\ \rightarrow\ -k\cdot\frac{u_{nr,nc+1}-u_{nr,nc-1}}{2\Delta x}\cdot(+1)=hr\cdot\left(u_{nr,nc}-ur\right)\ \rightarrow\ u_{nr,nc+1}=u_{nr,nc-1}-\frac{2\Delta x\cdot hr}{k}\cdot u_{nr,nc}+\frac{2\Delta x\cdot hr}{k}\cdot ur\ \Rightarrow$$

$$-u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,nc}^{m+1}-\left(u_{nr,nc-1}^{m+1}-\frac{2\Delta x\cdot hr}{k}\cdot u_{nr,nc}^{m+1}+\frac{2\Delta x\cdot hr}{k}\cdot ur\right)=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}\ \Rightarrow$$

$$\boxed{(i=nr)\quad(j=nc)}\ \rightarrow\ \boxed{-2u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}+\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{nr,nc}^{m+1}=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2-qt\cdot f_x\cdot\frac{2\Delta y}{k}+\frac{2\Delta x\cdot hr}{k}\cdot ur}$$

1.3.3 – Condição de contorno de troca de calor por convecção no bordo superior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y=ht\cdot(u-ut)\rightarrow -k\cdot\frac{u_{nr+1,j}-u_{nr-1,j}}{2\Delta y}\cdot(+1)=ht\cdot\left(u_{nr,j}-ut\right)\rightarrow\ u_{nr+1,j}=u_{nr-1,j}-\frac{2\Delta y\cdot ht}{k}\cdot u_{nr,j}+\frac{2\Delta y\cdot ht}{k}\cdot ut$$

$$-u_{nr,j-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,j}^{m+1}-u_{nr,j+1}^{m+1}=f_x\cdot u_{nr-1,j}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,j}^{m}+f_x\cdot u_{nr+1,j}^{m}+\frac{s_{nr,j}^{m}}{k\cdot e}\cdot\Delta x^2\ \Rightarrow$$

$$-u_{nr,j-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,j}^{m+1}-u_{nr,j+1}^{m+1}=f_x\cdot u_{nr-1,j}^{m}+\left(\frac{1}{r_x}-2f_x\right)\cdot u_{nr,j}^{m}+f_x\cdot\left(u_{nr-1,j}^{m}-\frac{2\Delta y\cdot ht}{k}\cdot u_{nr,j}^{m}+\frac{2\Delta y\cdot ht}{k}\cdot ut\right)+\frac{s_{nr,j}^{m}}{k\cdot e}\cdot\Delta x^2 \quad\Rightarrow$$

$$\boxed{(i=nr)\quad (j=2:nc-1)}\rightarrow\boxed{-u_{nr,j-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,j}^{m+1}-u_{nr,j+1}^{m+1}=2f_x\cdot u_{nr-1,j}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,j}^{m}+\frac{s_{nr,j}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut}$$

1.3.3.1 – Condições de contorno no bordo esquerdo

1.3.3.1.1 – Condição de contorno de temperatura constante prescrita no bordo esquerdo

$$\boxed{(i=nr)\quad (j=1)}\quad\rightarrow\quad\boxed{u_{nr,1}=ul}$$

1.3.3.1.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo esquerdo

$$qx=-k\cdot\frac{du}{dx}\cdot n_x\;\rightarrow\;qx_{nr,1}\cong-k\cdot\frac{u_{nr,2}-u_{nr,0}}{2\Delta x}\cdot(-1)=ql\;\rightarrow\quad u_{nr,0}=u_{nr,2}-ql\cdot\frac{2\Delta x}{k}$$

$$-u_{nr,0}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,1}^{m+1}-u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut\quad\Rightarrow$$

$$-\left(u_{nr,2}^{m+1}-ql\cdot\frac{2\Delta x}{k}\right)+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,1}^{m+1}-u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut\quad\Rightarrow$$

$$\boxed{(i=nr)\quad (j=1)}\quad\rightarrow\quad\boxed{\left(2+\frac{1}{r_x}\right)\cdot u_{nr,1}^{m+1}-2u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut-ql\cdot\frac{2\Delta x}{k}}$$

1.3.3.1.3 – Condição de contorno de troca de calor por convecção no bordo esquerdo

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hl\cdot(u-ul)\;\rightarrow\;-k\cdot\frac{u_{nr,2}-u_{nr,0}}{2\Delta x}\cdot(-1)=hl\cdot\left(u_{nr,1}-ul\right)\;\rightarrow\quad u_{nr,0}=-\frac{2\Delta x\cdot hl}{k}\cdot u_{nr,1}+u_{nr,2}+\frac{2\Delta x\cdot hl}{k}\cdot ul\quad\Rightarrow$$

$$-\left(-\frac{2\Delta x\cdot hl}{k}\cdot u_{nr,1}^{m+1}+u_{nr,2}^{m+1}+\frac{2\Delta x\cdot hl}{k}\cdot ul\right)+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,1}^{m+1}-u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut\quad\Rightarrow$$

$$\boxed{(i=nr)\quad (j=1)}\quad\rightarrow\quad\boxed{\left(2+\frac{1}{r_x}+\frac{2\Delta x\cdot hl}{k}\right)\cdot u_{nr,1}^{m+1}-2u_{nr,2}^{m+1}=2f_x\cdot u_{nr-1,1}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,1}^{m}+\frac{s_{nr,1}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut+\frac{2\Delta x\cdot hl}{k}\cdot ul}$$

1.3.3.2 – Condições de contorno no bordo direito

1.3.3.2.1 – Condição de contorno de temperatura constante prescrita no bordo direito

$$\boxed{(i=nr)\quad (j=nc)}\quad\rightarrow\quad\boxed{u_{nr,nc}=ur}$$

1.3.3.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo direito

$$qx=-k\cdot\frac{du}{dx}\cdot n_x\;\rightarrow\;qx_{nr,nc}\cong-k\cdot\frac{u_{nr,nc+1}-u_{nr,nc-1}}{2\Delta x}\cdot(+1)=qr\;\rightarrow\quad u_{nr,nc+1}=u_{nr,nc-1}-qr\cdot\frac{2\Delta x}{k}$$

$$-u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,nc}^{m+1}-u_{nr,nc+1}^{m+1}=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut\quad\Rightarrow$$

$$-u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,nc}^{m+1}-\left(u_{nr,nc-1}^{m+1}-qr\cdot\frac{2\Delta x}{k}\right)=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut\quad\Rightarrow$$

$$\boxed{(i=nr)\quad (j=nc)}\quad\rightarrow\quad\boxed{-2u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,nc}^{m+1}=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut-qr\cdot\frac{2\Delta x}{k}}$$

1.3.3.2.3 – Condição de contorno de troca de calor por convecção no bordo direito

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hr\cdot(u-ur)\;\rightarrow\;-k\cdot\frac{u_{nr,nc+1}-u_{nr,nc-1}}{2\Delta x}\cdot(+1)=hr\cdot\left(u_{nr,nc}-ur\right)\;\rightarrow\quad u_{nr,nc+1}=u_{nr,nc-1}-\frac{2\Delta x\cdot hr}{k}\cdot u_{nr,nc}+\frac{2\Delta x\cdot hr}{k}\cdot ur\quad\Rightarrow$$

$$-u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}\right)\cdot u_{nr,nc}^{m+1}-\left(u_{nr,nc-1}^{m+1}-\frac{2\Delta x\cdot hr}{k}\cdot u_{nr,nc}^{m+1}+\frac{2\Delta x\cdot hr}{k}\cdot ur\right)=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut\quad\Rightarrow$$

$$\boxed{(i=nr)\quad (j=nc)}\quad\rightarrow\quad\boxed{-2u_{nr,nc-1}^{m+1}+\left(2+\frac{1}{r_x}+\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{nr,nc}^{m+1}=2f_x\cdot u_{nr-1,nc}^{m}+\left(\frac{1}{r_x}-2f_x-f_x\cdot\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,nc}^{m}+\frac{s_{nr,nc}^{m}}{k\cdot e}\cdot\Delta x^2+f_x\cdot\frac{2\Delta y\cdot ht}{k}\cdot ut+\frac{2\Delta x\cdot hr}{k}\cdot ur}$$

2 – Resolvendo em y: (segundo sub-passo no tempo m+2)

$$\frac{u_{i,j-1}^{m+1}-2u_{i,j}^{m+1}+u_{i,j+1}^{m+1}}{\Delta x^2}+\frac{u_{i-1,j}^{m+2}-2u_{i,j}^{m+2}+u_{i+1}^{m+2}}{\Delta y^2}-\frac{u_{i,j}^{m+2}}{\alpha\cdot\Delta t}\cdot=-\frac{u_{i,j}^{m+1}}{\alpha\cdot\Delta t}-\frac{s_{i,j}^{m+1}}{k\cdot e}$$

↑ considerado igual ao do sub - passo anterior; ↑ desconhecido; ↑ desconhecido

$$\Rightarrow\quad u_{i-1,j}^{m+2}-2u_{i,j}^{m+2}+u_{i+1,j}^{m+2}-\frac{u_{i,j}^{m+2}}{\alpha\cdot\Delta t}\cdot\Delta y^2=-\frac{\left(u_{i,j-1}^{m+1}-2u_{i,j}^{m+1}+u_{i,j+1}^{m+1}\right)}{\Delta x^2}\cdot\Delta y^2-\frac{u_{i,j}^{m+1}}{\alpha\cdot\Delta t}\cdot\Delta y^2-\frac{s_{i,j}^{m+1}}{k\cdot e}\cdot\Delta y^2$$

2.1 – Solução da coluna genérica (j = 2:nc–1)

2.1.1 – Caso geral (células do interior da coluna)

Adotando $\boxed{r_y=\frac{\alpha\cdot\Delta t}{\Delta y^2}}$, $\boxed{f_y=\frac{1}{f_x}=\frac{\Delta y^2}{\Delta x^2}}$ e $[\times(-1)]$ $\Rightarrow$

$$\boxed{(i=2:nr-1)\,(j=2:nc-1)}\rightarrow\boxed{-u_{i-1,j}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,j}^{m+2}-u_{i+1,j}^{m+2}=f_y\cdot u_{i,j-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{i,j}^{m+1}+f_y\cdot u_{i,j+1}^{m+1}+\frac{s_{i,j}^{m+1}}{k\cdot e}\cdot\Delta y^2}$$

2.1.2 – Condições de contorno no bordo inferior

2.1.2.1 – Condição de contorno de temperatura constante prescrita no bordo inferior

$$\boxed{(i=1)\quad(j=2:nc-1)}\ \rightarrow\ \boxed{u_{1,j}=ub}$$

2.1.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo inferior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y\ \rightarrow\ qy_{1,j}\cong-k\cdot\frac{u_{2,j}-u_{0,j}}{2\Delta y}\cdot(-1)=qb\ \rightarrow\ u_{0,j}=u_{2,j}-qb\cdot\frac{2\Delta y}{k}$$

$$-u_{0,j}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{1,j}^{m+2}-u_{2,j}^{m+2}=f_y\cdot u_{1,j-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,j}^{m+1}+f_y\cdot u_{1,j+1}^{m+1}+\frac{s_{1,j}^{m+1}}{k\cdot e}\cdot\Delta y^2\quad\Rightarrow$$

$$-\left(u_{2,j}^{m+2}-qb\cdot\frac{2\Delta y}{k}\right)+\left(2+\frac{1}{r_y}\right)\cdot u_{1,j}^{m+2}-u_{2,j}^{m+2}=f_y\cdot u_{1,j-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,j}^{m+1}+f_y\cdot u_{1,j+1}^{m+1}+\frac{s_{1,j}^{m+1}}{k\cdot e}\cdot\Delta y^2\quad\Rightarrow$$

$$\boxed{(i=1)\ \ (j=2:nc-1)}\rightarrow\boxed{\left(2+\frac{1}{r_y}\right)\cdot u_{1,j}^{m+2}-2u_{2,j}^{m+2}=f_y\cdot u_{1,j-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,j}^{m+1}+f_y\cdot u_{1,j+1}^{m+1}+\frac{s_{1,j}^{m+1}}{k\cdot e}\cdot\Delta y^2-qb\cdot\frac{2\Delta y}{k}}$$

2.1.2.3 – Condição de contorno de troca de calor por convecção no bordo inferior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y=hb\cdot(u-ub)\rightarrow\ -k\cdot\frac{u_{2,j}-u_{0,j}}{2\Delta y}\cdot(-1)=hb\cdot\left(u_{1,j}-ub\right)\ \rightarrow\ u_{0,j}=-\frac{2\Delta y\cdot hb}{k}\cdot u_{1,j}+u_{2,j}+\frac{2\Delta y\cdot hb}{k}\cdot ub\quad\Rightarrow$$

$$-\left(-\frac{2\Delta y\cdot hb}{k}\cdot u_{1,j}^{m+2}+u_{2,j}^{m+2}+\frac{2\Delta y\cdot hb}{k}\cdot ub\right)+\left(2+\frac{1}{r_y}\right)\cdot u_{1,j}^{m+2}-u_{2,j}^{m+2}=f_y\cdot u_{1,j-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,j}^{m+1}+f_y\cdot u_{1,j+1}^{m+1}+\frac{s_{1,j}^{m+1}}{k\cdot e}\cdot\Delta y^2\quad\Rightarrow$$

$$\boxed{(i=1)\ \ (j=2:nc-1)}\rightarrow\boxed{\left(2+\frac{1}{r_y}+\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,j}^{m+2}-2u_{2,j}^{m+2}=f_y\cdot u_{1,j-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,j}^{m+1}+f_y\cdot u_{1,j+1}^{m+1}+\frac{s_{1,j}^{m+1}}{k\cdot e}\cdot\Delta y^2+\frac{2\Delta y\cdot hb}{k}\cdot ub}$$

2.1.3 – Condições de contorno no bordo superior

2.1.3.1 – Condição de contorno de temperatura constante prescrita no bordo superior

$$\boxed{(i=nr)\quad(j=2:nc-1)}\ \rightarrow\ \boxed{u_{nr,j}=ut}$$

2.1.3.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo superior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y \rightarrow qy_{nr,j} \cong -k \cdot \frac{u_{nr+1,j} - u_{nr-1,j}}{2\Delta y} \cdot (+1) = qt \rightarrow \quad u_{nr+1,j} = u_{nr-1,j} - qt \cdot \frac{2\Delta y}{k}$$

$$-u_{nr-1,j}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{nr,j}^{m+2} - u_{nr+1,j}^{m+2} = f_y \cdot u_{nr,j-1}^{m+1} + \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{nr,j}^{m+1} + f_y \cdot u_{nr,j+1}^{m+1} + \frac{s_{nr,j}^{m+1}}{k \cdot e} \cdot \Delta y^2 \quad \Rightarrow$$

$$-u_{nr-1,j}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{nr,j}^{m+2} - \left(u_{nr-1,j}^{m+2} - qt \cdot \frac{2\Delta y}{k}\right) = f_y \cdot u_{nr,j-1}^{m+1} + \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{nr,j}^{m+1} + f_y \cdot u_{nr,j+1}^{m+1} + \frac{s_{nr,j}^{m+1}}{k \cdot e} \cdot \Delta y^2 \quad \Rightarrow$$

$$\boxed{(i = nr) \quad (j = 2 : nc-1)} \rightarrow \boxed{-2u_{nr-1,j}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{nr,j}^{m+2} = f_y \cdot u_{nr,j-1}^{m+1} + \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{nr,j}^{m+1} + f_y \cdot u_{nr,j+1}^{m+1} + \frac{s_{nr,j}^{m+1}}{k \cdot e} \cdot \Delta y^2 - qt \cdot \frac{2\Delta y}{k}}$$

2.1.3.3 – Condição de contorno de troca de calor por convecção no bordo superior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y = ht \cdot (u - ut) \rightarrow -k \cdot \frac{u_{nr+1,j} - u_{nr-1,j}}{2\Delta y} \cdot (+1) = ht \cdot \left(u_{nr,j} - ut\right) \rightarrow \quad u_{nr+1,j} = u_{nr-1,j} - \frac{2\Delta y \cdot ht}{k} \cdot u_{nr,j} + \frac{2\Delta y \cdot ht}{k} \cdot ut \quad \Rightarrow$$

$$-u_{nr-1,j}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{nr,j}^{m+2} - \left(u_{nr-1,j}^{m+2} - \frac{2\Delta y \cdot ht}{k} \cdot u_{nr,j}^{m+2} + \frac{2\Delta y \cdot ht}{k} \cdot ut\right) = f_y \cdot u_{nr,j-1}^{m+1} + \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{nr,j}^{m+1} + f_y \cdot u_{nr,j+1}^{m+1} + \frac{s_{nr,j}^{m+1}}{k \cdot e} \cdot \Delta y^2 \quad \Rightarrow$$

$$\boxed{(i = nr) \quad (j = 2 : nc-1)} \rightarrow \boxed{-2u_{nr-1,j}^{m+2} + \left(2 + \frac{1}{r_y} + \frac{2\Delta y \cdot ht}{k}\right) \cdot u_{nr,j}^{m+2} = f_y \cdot u_{nr,j-1}^{m+1} + \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{nr,j}^{m+1} + f_y \cdot u_{nr,j+1}^{m+1} + \frac{s_{nr,j}^{m+1}}{k \cdot e} \cdot \Delta y^2 + \frac{2\Delta y \cdot ht}{k} \cdot ut}$$

2.2 – Solução da coluna da esquerda ($j = 1$)

2.2.1 – Condição de contorno de temperatura constante prescrita no bordo esquerdo

$$\boxed{(i = 1 : nr) \quad (j = 1)} \quad \rightarrow \quad \boxed{u_{i,1} = ul}$$

2.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo esquerdo

$$qx = -k \cdot \frac{du}{dx} \cdot n_x \quad \rightarrow \quad qx_{i,1} \cong -k \cdot \frac{u_{i,2} - u_{i,0}}{2\Delta x} \cdot (-1) = ql \quad \rightarrow \quad u_{i,0} = u_{i,2} - ql \cdot \frac{2\Delta x}{k}$$

$$-u_{i-1,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{i,1}^{m+2} - u_{i+1,1}^{m+2} = f_y \cdot u_{i,0}^{m+1} + \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{i,1}^{m+1} + f_y \cdot u_{i,2}^{m+1} + \frac{s_{i,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 \quad \Rightarrow$$

$$-u_{i-1,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{i,1}^{m+2} - u_{i+1,1}^{m+2} = f_y \cdot \left(u_{i,2}^{m+1} - ql \cdot \frac{2\Delta x}{k}\right) + \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{i,1}^{m+1} + f_y \cdot u_{i,2}^{m+1} + \frac{s_{i,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 \quad \Rightarrow$$

$$\boxed{(i = 2 : nr-1) \quad (j = 1)} \quad \rightarrow \quad \boxed{-u_{i-1,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{i,1}^{m+2} - u_{i+1,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{i,1}^{m+1} + 2f_y \cdot u_{i,2}^{m+1} + \frac{s_{i,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 - ql \cdot f_y \cdot \frac{2\Delta x}{k}}$$

2.2.2.1 – Condições de contorno no bordo inferior

2.2.2.1.1 – Condição de contorno de temperatura constante prescrita no bordo inferior

$$\boxed{(i = 1) \quad (j = 1)} \quad \rightarrow \quad \boxed{u_{1,1} = ub}$$

2.2.2.1.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo inferior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y \rightarrow qy_{1,1} \cong -k \cdot \frac{u_{2,1} - u_{0,1}}{2\Delta y} \cdot (-1) = qb \rightarrow \quad u_{0,1} = u_{2,1} - qb \cdot \frac{2\Delta y}{k}$$

$$-u_{0,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{1,1}^{m+2} - u_{2,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{1,1}^{m+1} + 2f_y \cdot u_{1,2}^{m+1} + \frac{s_{1,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 - ql \cdot f_y \cdot \frac{2\Delta x}{k} \quad \Rightarrow$$

$$-\left(u_{2,1}^{m+2} - qb \cdot \frac{2\Delta y}{k}\right) + \left(2 + \frac{1}{r_y}\right) \cdot u_{1,1}^{m+2} - u_{2,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y\right) \cdot u_{1,1}^{m+1} + 2f_y \cdot u_{1,2}^{m+1} + \frac{s_{1,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 - ql \cdot f_y \cdot \frac{2\Delta x}{k} \quad \Rightarrow$$

$$\boxed{(i=1)\quad (j=1)} \rightarrow \boxed{\left(2+\frac{1}{r_y}\right)\cdot u_{1,1}^{m+2}-2u_{2,1}^{m+2}=\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,1}^{m+1}+2f_y\cdot u_{1,2}^{m+1}+\frac{s_{1,1}^{m+1}}{k\cdot e}\cdot\Delta y^2-ql\cdot f_y\cdot\frac{2\Delta x}{k}-qb\cdot\frac{2\Delta y}{k}}$$

2.2.2.1.3 – Condição de contorno de troca de calor por convecção no bordo inferior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y=hb\cdot(u-ub)\rightarrow\ -k\cdot\frac{u_{2,1}-u_{0,1}}{2\Delta y}\cdot(-1)=hb\cdot(u_{1,1}-ub)\ \rightarrow\ \ u_{0,1}=-\frac{2\Delta y\cdot hb}{k}\cdot u_{1,1}+u_{2,1}+\frac{2\Delta y\cdot hb}{k}\cdot ub\ \Rightarrow$$

$$-\left(-\frac{2\Delta y\cdot hb}{k}\cdot u_{1,1}^{m+2}+u_{2,1}^{m+2}+\frac{2\Delta y\cdot hb}{k}\cdot ub\right)+\left(2+\frac{1}{r_y}\right)\cdot u_{1,1}^{m+2}-u_{2,1}^{m+2}=\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,1}^{m+1}+2f_y\cdot u_{1,2}^{m+1}+\frac{s_{1,1}^{m+1}}{k\cdot e}\cdot\Delta y^2-ql\cdot f_y\cdot\frac{2\Delta x}{k}\ \Rightarrow$$

$$\boxed{(i=1)\quad (j=1)} \rightarrow \boxed{\left(2+\frac{1}{r_y}+\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,1}^{m+2}-2u_{2,1}^{m+2}=\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,1}^{m+1}+2f_y\cdot u_{1,2}^{m+1}+\frac{s_{1,1}^{m+1}}{k\cdot e}\cdot\Delta y^2-ql\cdot f_y\cdot\frac{2\Delta x}{k}+\frac{2\Delta y\cdot hb}{k}\cdot ub}$$

2.2.2.2 – Condições de contorno no bordo superior

2.2.2.2.1 – Condição de contorno de temperatura constante prescrita no bordo superior

$$\boxed{(i=nr)\quad (j=1)} \rightarrow \boxed{u_{nr,1}=ut}$$

2.2.2.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo superior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y\rightarrow qy_{nr,1}\cong-k\cdot\frac{u_{nr+1,1}-u_{nr-1,1}}{2\Delta y}\cdot(+1)=qt\ \rightarrow\ \ u_{nr+1,1}=u_{nr-1,1}-qt\cdot\frac{2\Delta y}{k}$$

$$-u_{nr-1,1}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,1}^{m+2}-u_{nr+1,1}^{m+2}=\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,1}^{m+1}+2f_y\cdot u_{nr,2}^{m+1}+\frac{s_{nr,1}^{m+1}}{k\cdot e}\cdot\Delta y^2-ql\cdot f_y\cdot\frac{2\Delta x}{k}\ \Rightarrow$$

$$-u_{nr-1,1}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,1}^{m+2}-\left(u_{nr-1,1}^{m+2}-qt\cdot\frac{2\Delta y}{k}\right)=\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,1}^{m+1}+2f_y\cdot u_{nr,2}^{m+1}+\frac{s_{nr,1}^{m+1}}{k\cdot e}\cdot\Delta y^2-ql\cdot f_y\cdot\frac{2\Delta x}{k}\ \Rightarrow$$

$$\boxed{(i=nr)\quad (j=1)} \rightarrow \boxed{-2u_{nr-1,1}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,1}^{m+2}=\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,1}^{m+1}+2f_y\cdot u_{nr,2}^{m+1}+\frac{s_{nr,1}^{m+1}}{k\cdot e}\cdot\Delta y^2-ql\cdot f_y\cdot\frac{2\Delta x}{k}-qt\cdot\frac{2\Delta y}{k}}$$

2.2.2.2.3 – Condição de contorno de troca de calor por convecção no bordo superior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y=ht\cdot(u-ut)\rightarrow-k\cdot\frac{u_{nr+1,1}-u_{nr-1,1}}{2\Delta y}\cdot(+1)=ht\cdot(u_{nr,1}-ut)\rightarrow\ \ u_{nr+1,1}=u_{nr-1,1}-\frac{2\Delta y\cdot ht}{k}\cdot u_{nr,1}+\frac{2\Delta y\cdot ht}{k}\cdot ut\ \Rightarrow$$

$$-u_{nr-1,1}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,1}^{m+2}-\left(u_{nr-1,1}^{m+2}-\frac{2\Delta y\cdot ht}{k}\cdot u_{nr,1}^{m+2}+\frac{2\Delta y\cdot ht}{k}\cdot ut\right)=\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,1}^{m+1}+2f_y\cdot u_{nr,2}^{m+1}+\frac{s_{nr,1}^{m+1}}{k\cdot e}\cdot\Delta y^2-ql\cdot f_y\cdot\frac{2\Delta x}{k}\ \Rightarrow$$

$$\boxed{(i=nr)\quad (j=1)} \rightarrow \boxed{-2u_{nr-1,1}^{m+2}+\left(2+\frac{1}{r_y}+\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,1}^{m+2}=\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,1}^{m+1}+2f_y\cdot u_{nr,2}^{m+1}+\frac{s_{nr,1}^{m+1}}{k\cdot e}\cdot\Delta y^2-ql\cdot f_y\cdot\frac{2\Delta x}{k}+\frac{2\Delta y\cdot ht}{k}\cdot ut}$$

2.2.3 – Condição de contorno de troca de calor por convecção no bordo esquerdo

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hl\cdot(u-ul)\ \rightarrow\ -k\cdot\frac{u_{i,2}-u_{i,0}}{2\Delta x}\cdot(-1)=hl\cdot(u_{i,1}-ul)\ \rightarrow\ \ u_{i,0}=-\frac{2\Delta x\cdot hl}{k}\cdot u_{i,1}+u_{i,2}+\frac{2\Delta x\cdot hl}{k}\cdot ul$$

$$-u_{i-1,1}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,1}^{m+2}-u_{i+1,1}^{m+2}=f_y\cdot u_{i,0}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{i,1}^{m+1}+f_y\cdot u_{i,2}^{m+1}+\frac{s_{i,1}^{m+1}}{k\cdot e}\cdot\Delta y^2\ \Rightarrow$$

$$-u_{i-1,1}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,1}^{m+2}-u_{i+1,1}^{m+2}=f_y\cdot\left(-\frac{2\Delta x\cdot hl}{k}\cdot u_{i,1}^{m+1}+u_{i,2}^{m+1}+\frac{2\Delta x\cdot hl}{k}\cdot ul\right)+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{i,1}^{m+1}+f_y\cdot u_{i,2}^{m+1}+\frac{s_{i,1}^{m+1}}{k\cdot e}\cdot\Delta y^2\ \Rightarrow$$

$$\boxed{(i=2:nr-1)\quad (j=1)} \rightarrow \boxed{-u_{i-1,1}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,1}^{m+2}-u_{i+1,1}^{m+2}=\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hl}{k}\right)\cdot u_{i,1}^{m+1}+2f_y\cdot u_{i,2}^{m+1}+\frac{s_{i,1}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hl}{k}\cdot ul}$$

2.2.3.1 – Condições de contorno no bordo inferior

2.2.3.1.1 – Condição de contorno de temperatura constante prescrita no bordo inferior

$$\boxed{(i=1)\quad (j=1)} \rightarrow \boxed{u_{1,1}=ub}$$

2.2.3.1.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo inferior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y \rightarrow qy_{1,1} \cong -k \cdot \frac{u_{2,1} - u_{0,1}}{2\Delta y} \cdot (-1) = qb \rightarrow \quad u_{0,1} = u_{2,1} - qb \cdot \frac{2\Delta y}{k}$$

$$-u_{0,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{1,1}^{m+2} - u_{2,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{1,1}^{m+1} + 2f_y \cdot u_{1,2}^{m+1} + \frac{s_{1,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul \quad \Rightarrow$$

$$-\left(u_{2,1}^{m+2} - qb \cdot \frac{2\Delta y}{k}\right) + \left(2 + \frac{1}{r_y}\right) \cdot u_{1,1}^{m+2} - u_{2,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{1,1}^{m+1} + 2f_y \cdot u_{1,2}^{m+1} + \frac{s_{1,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul \quad \Rightarrow$$

$$\boxed{(i=1) \quad (j=1)} \quad \rightarrow \quad \boxed{\left(2 + \frac{1}{r_y}\right) \cdot u_{1,1}^{m+2} - 2u_{2,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{1,1}^{m+1} + 2f_y \cdot u_{1,2}^{m+1} + \frac{s_{1,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul - qb \cdot \frac{2\Delta y}{k}}$$

2.2.3.1.3 – Condição de contorno de troca de calor por convecção no bordo inferior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y = hb \cdot (u - ub) \rightarrow \; -k \cdot \frac{u_{2,1} - u_{0,1}}{2\Delta y} \cdot (-1) = hb \cdot (u_{1,1} - ub) \rightarrow \quad u_{0,1} = -\frac{2\Delta y \cdot hb}{k} \cdot u_{1,1} + u_{2,1} + \frac{2\Delta y \cdot hb}{k} \cdot ub \quad \Rightarrow$$

$$-\left(-\frac{2\Delta y \cdot hb}{k} \cdot u_{1,1}^{m+2} + u_{2,1}^{m+2} + \frac{2\Delta y \cdot hb}{k} \cdot ub\right) + \left(2 + \frac{1}{r_y}\right) \cdot u_{1,1}^{m+2} - u_{2,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{1,1}^{m+1} + 2f_y \cdot u_{1,2}^{m+1} + \frac{s_{1,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul \quad \Rightarrow$$

$$\boxed{(i=1) \quad (j=1)} \quad \rightarrow \quad \boxed{\left(2 + \frac{1}{r_y} + \frac{2\Delta y \cdot hb}{k}\right) \cdot u_{1,1}^{m+2} - 2u_{2,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{1,1}^{m+1} + 2f_y \cdot u_{1,2}^{m+1} + \frac{s_{1,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul + \frac{2\Delta y \cdot hb}{k} \cdot ub}$$

2.2.3.2 – Condições de contorno no bordo superior

2.2.3.2.1 – Condição de contorno de temperatura constante prescrita no bordo superior

$$\boxed{(i=nr) \quad (j=1)} \quad \rightarrow \quad \boxed{u_{nr,1} = ut}$$

2.2.3.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo superior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y \rightarrow qy_{nr,1} \cong -k \cdot \frac{u_{nr+1,1} - u_{nr-1,1}}{2\Delta y} \cdot (+1) = qt \rightarrow \quad u_{nr+1,1} = u_{nr-1,1} - qt \cdot \frac{2\Delta y}{k}$$

$$-u_{nr-1,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{nr,1}^{m+2} - u_{nr+1,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{nr,1}^{m+1} + 2f_y \cdot u_{nr,2}^{m+1} + \frac{s_{nr,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul \quad \Rightarrow$$

$$-u_{nr-1,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{nr,1}^{m+2} - \left(u_{nr-1,1}^{m+2} - qt \cdot \frac{2\Delta y}{k}\right) = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{nr,1}^{m+1} + 2f_y \cdot u_{nr,2}^{m+1} + \frac{s_{nr,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul \quad \Rightarrow$$

$$\boxed{(i=nr) \quad (j=1)} \quad \rightarrow \quad \boxed{-2u_{nr-1,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{nr,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{nr,1}^{m+1} + 2f_y \cdot u_{nr,2}^{m+1} + \frac{s_{nr,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul - qt \cdot \frac{2\Delta y}{k}}$$

2.2.3.2.3 – Condição de contorno de troca de calor por convecção no bordo superior

$$qy = -k \cdot \frac{du}{dy} \cdot n_y = ht \cdot (u - ut) \rightarrow -k \cdot \frac{u_{nr+1,1} - u_{nr-1,1}}{2\Delta y} \cdot (+1) = ht \cdot (u_{nr,1} - ut) \rightarrow \quad u_{nr+1,1} = u_{nr-1,1} - \frac{2\Delta y \cdot ht}{k} \cdot u_{nr,1} + \frac{2\Delta y \cdot ht}{k} \cdot ut \quad \Rightarrow$$

$$-u_{nr-1,1}^{m+2} + \left(2 + \frac{1}{r_y}\right) \cdot u_{nr,1}^{m+2} - \left(u_{nr-1,1}^{m+2} - \frac{2\Delta y \cdot ht}{k} \cdot u_{nr,1}^{m+2} + \frac{2\Delta y \cdot ht}{k} \cdot ut\right) = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{nr,1}^{m+1} + 2f_y \cdot u_{nr,2}^{m+1} + \frac{s_{nr,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul \quad \Rightarrow$$

$$\boxed{(i=nr) \quad (j=1)} \quad \rightarrow \quad \boxed{-2u_{nr-1,1}^{m+2} + \left(2 + \frac{1}{r_y} + \frac{2\Delta y \cdot ht}{k}\right) \cdot u_{nr,1}^{m+2} = \left(\frac{1}{r_y} - 2f_y - f_y \cdot \frac{2\Delta x \cdot hl}{k}\right) \cdot u_{nr,1}^{m+1} + 2f_y \cdot u_{nr,2}^{m+1} + \frac{s_{nr,1}^{m+1}}{k \cdot e} \cdot \Delta y^2 + f_y \cdot \frac{2\Delta x \cdot hl}{k} \cdot ul + \frac{2\Delta y \cdot ht}{k} \cdot ut}$$

2.3 – Solução da coluna da direita ($j = nc$)

2.3.1 – Condição de contorno de temperatura constante prescrita no bordo direito

$$\boxed{(i=1:nr) \quad (j=nc)} \quad \rightarrow \quad \boxed{u_{i,nc} = ur}$$

2.3.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo direito

$$qx = -k \cdot \frac{du}{dx} \cdot n_x \;\rightarrow\; qx_{i,nc} \cong -k \cdot \frac{u_{i,nc+1} - u_{i,nc-1}}{2\Delta x} \cdot (+1) = qr \;\rightarrow\; u_{i,nc+1} = u_{i,nc-1} - qr \cdot \frac{2\Delta x}{k}$$

$$-u_{i-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,nc}^{m+2}-u_{i+1,nc}^{m+2}=f_y\cdot u_{i,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{i,nc}^{m+1}+f_y\cdot u_{i,nc+1}^{m+1}+\frac{s_{i,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2 \quad\Rightarrow$$

$$-u_{i-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,nc}^{m+2}-u_{i+1,nc}^{m+2}=f_y\cdot u_{i,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{i,nc}^{m+1}+f_y\cdot\left(u_{i,nc-1}^{m+1}-qr\cdot\frac{2\Delta x}{k}\right)+\frac{s_{i,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2 \quad\Rightarrow$$

$$\boxed{(i=2:nr-1)\quad(j=nc)}\rightarrow\boxed{-u_{i-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,nc}^{m+2}-u_{i+1,nc}^{m+2}=2f_y\cdot u_{i,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{i,nc}^{m+1}+\frac{s_{i,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}}$$

2.3.2.1 – Condições de contorno no bordo inferior

2.3.2.1.1 – Condição de contorno de temperatura constante prescrita no bordo inferior

$$\boxed{(i=1)\quad(j=nc)}\quad\rightarrow\quad\boxed{u_{1,nc}=ub}$$

2.3.2.1.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo inferior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y\;\rightarrow\;qy_{1,nc}\cong-k\cdot\frac{u_{2,nc}-u_{0,nc}}{2\Delta y}\cdot(-1)=qb\;\rightarrow\quad u_{0,nc}=u_{2,nc}-qb\cdot\frac{2\Delta y}{k}$$

$$-u_{0,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{1,nc}^{m+2}-u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}\quad\Rightarrow$$

$$-\left(u_{2,nc}^{m+2}-qb\cdot\frac{2\Delta y}{k}\right)+\left(2+\frac{1}{r_y}\right)\cdot u_{1,nc}^{m+2}-u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}\quad\Rightarrow$$

$$\boxed{(i=1)\quad(j=nc)}\quad\rightarrow\quad\boxed{\left(2+\frac{1}{r_y}\right)\cdot u_{1,nc}^{m+2}-2u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}-qb\cdot\frac{2\Delta y}{k}}$$

2.3.2.1.3 – Condição de contorno de troca de calor por convecção no bordo inferior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y=hb\cdot(u-ub)\rightarrow\;-k\cdot\frac{u_{2,nc}-u_{0,nc}}{2\Delta y}\cdot(-1)=hb\cdot\left(u_{1,nc}-ub\right)\;\rightarrow\quad u_{0,nc}=-\frac{2\Delta y\cdot hb}{k}\cdot u_{1,nc}+u_{2,nc}+\frac{2\Delta y\cdot hb}{k}\cdot ub\quad\Rightarrow$$

$$-\left(-\frac{2\Delta y\cdot hb}{k}\cdot u_{1,nc}^{m+2}+u_{2,nc}^{m+2}+\frac{2\Delta y\cdot hb}{k}\cdot ub\right)+\left(2+\frac{1}{r_y}\right)\cdot u_{1,nc}^{m+2}-u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}\quad\Rightarrow$$

$$\boxed{(i=1)\quad(j=nc)}\quad\rightarrow\quad\boxed{\left(2+\frac{1}{r_y}+\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,nc}^{m+2}-2u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}+\frac{2\Delta y\cdot hb}{k}\cdot ub}$$

2.3.2.2 – Condições de contorno no bordo superior

2.3.2.2.1 – Condição de contorno de temperatura constante prescrita no bordo superior

$$\boxed{(i=nr)\quad(j=nc)}\quad\rightarrow\quad\boxed{u_{nr,nc}=ut}$$

2.3.2.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo superior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y\rightarrow qy_{nr,nc}\cong-k\cdot\frac{u_{nr+1,nc}-u_{nr-1,nc}}{2\Delta y}\cdot(+1)=qt\;\rightarrow\quad u_{nr+1,nc}=u_{nr-1,nc}-qt\cdot\frac{2\Delta y}{k}$$

$$-u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,nc}^{m+2}-u_{nr+1,nc}^{m+2}=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}\quad\Rightarrow$$

$$-u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,nc}^{m+2}-\left(u_{nr-1,nc}^{m+2}-qt\cdot\frac{2\Delta y}{k}\right)=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}\quad\Rightarrow$$

$$\boxed{(i=nr)\quad(j=nc)}\quad\rightarrow\quad\boxed{-2u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,nc}^{m+2}=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}-qt\cdot\frac{2\Delta y}{k}}$$

2.3.2.2.3 – Condição de contorno de troca de calor por convecção no bordo superior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y=ht\cdot(u-ut)\rightarrow-k\cdot\frac{u_{nr+1,nc}-u_{nr-1,nc}}{2\Delta y}\cdot(+1)=ht\cdot\left(u_{nr,nc}-ut\right)\rightarrow\quad u_{nr+1,nc}=u_{nr-1,nc}-\frac{2\Delta y\cdot ht}{k}\cdot u_{nr,nc}+\frac{2\Delta y\cdot ht}{k}\cdot ut\quad\Rightarrow$$

$$-u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,nc}^{m+2}-\left(u_{nr-1,nc}^{m+2}-\frac{2\Delta y\cdot ht}{k}\cdot u_{nr,nc}^{m+2}+\frac{2\Delta y\cdot ht}{k}\cdot ut\right)=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}\ \Rightarrow$$

$$\boxed{(i=nr)\quad(j=nc)}\ \rightarrow\ \boxed{-2u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}+\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,nc}^{m+2}=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2-qr\cdot f_y\cdot\frac{2\Delta x}{k}+\frac{2\Delta y\cdot ht}{k}\cdot ut}$$

2.3.3 – Condição de contorno de troca de calor por convecção no bordo direito

$$qx=-k\cdot\frac{du}{dx}\cdot n_x=hr\cdot(u-ur)\ \rightarrow\ -k\cdot\frac{u_{i,nc+1}-u_{i,nc-1}}{2\Delta x}\cdot(+1)=hr\cdot(u_{i,nc}-ur)\ \rightarrow\ \ u_{i,nc+1}=u_{i,nc-1}-\frac{2\Delta x\cdot hr}{k}\cdot u_{i,nc}+\frac{2\Delta x\cdot hr}{k}\cdot ur$$

$$-u_{i-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,nc}^{m+2}-u_{i+1,nc}^{m+2}=f_y\cdot u_{i,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{i,nc}^{m+1}+f_y\cdot u_{i,nc+1}^{m+1}+\frac{s_{i,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2\ \Rightarrow$$

$$-u_{i-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,nc}^{m+2}-u_{i+1,nc}^{m+2}=f_y\cdot u_{i,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y\right)\cdot u_{i,nc}^{m+1}+f_y\cdot\left(u_{i,nc-1}^{m+1}-\frac{2\Delta x\cdot hr}{k}\cdot u_{i,nc}^{m+1}+\frac{2\Delta x\cdot hr}{k}\cdot ur\right)+\frac{s_{i,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2\ \Rightarrow$$

$$\boxed{(i=2:nr-1)\quad(j=nc)}\rightarrow\boxed{-u_{i-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{i,nc}^{m+2}-u_{i+1,nc}^{m+2}=2f_y\cdot u_{i,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{i,nc}^{m+1}+\frac{s_{i,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur}$$

2.3.3.1 – Condições de contorno no bordo inferior

2.3.3.1.1 – Condição de contorno de temperatura constante prescrita no bordo inferior

$$\boxed{(i=1)\quad(j=nc)}\ \rightarrow\ \boxed{u_{1,nc}=ub}$$

2.3.3.1.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo inferior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y\ \rightarrow\ qy_{1,nc}\cong-k\cdot\frac{u_{2,nc}-u_{0,nc}}{2\Delta y}\cdot(-1)=qb\ \rightarrow\ \ u_{0,nc}=u_{2,nc}-qb\cdot\frac{2\Delta y}{k}$$

$$-u_{0,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{1,nc}^{m+2}-u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur\ \Rightarrow$$

$$-\left(u_{2,nc}^{m+2}-qb\cdot\frac{2\Delta y}{k}\right)+\left(2+\frac{1}{r_y}\right)\cdot u_{1,nc}^{m+2}-u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur\ \Rightarrow$$

$$\boxed{(i=1)\quad(j=nc)}\ \rightarrow\ \boxed{\left(2+\frac{1}{r_y}\right)\cdot u_{1,nc}^{m+2}-2u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur-qb\cdot\frac{2\Delta y}{k}}$$

2.3.3.1.3 – Condição de contorno de troca de calor por convecção no bordo inferior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y=hb\cdot(u-ub)\rightarrow\ -k\cdot\frac{u_{2,nc}-u_{0,nc}}{2\Delta y}\cdot(-1)=hb\cdot(u_{1,nc}-ub)\ \rightarrow\ \ u_{0,nc}=-\frac{2\Delta y\cdot hb}{k}\cdot u_{1,nc}+u_{2,nc}+\frac{2\Delta y\cdot hb}{k}\cdot ub\ \Rightarrow$$

$$-\left(-\frac{2\Delta y\cdot hb}{k}\cdot u_{1,nc}^{m+2}+u_{2,nc}^{m+2}+\frac{2\Delta y\cdot hb}{k}\cdot ub\right)+\left(2+\frac{1}{r_y}\right)\cdot u_{1,nc}^{m+2}-u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur\ \Rightarrow$$

$$\boxed{(i=1)\quad(j=nc)}\ \rightarrow\ \boxed{\left(2+\frac{1}{r_y}+\frac{2\Delta y\cdot hb}{k}\right)\cdot u_{1,nc}^{m+2}-2u_{2,nc}^{m+2}=2f_y\cdot u_{1,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{1,nc}^{m+1}+\frac{s_{1,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur+\frac{2\Delta y\cdot hb}{k}\cdot ub}$$

2.3.3.2 – Condições de contorno no bordo superior

2.3.3.2.1 – Condição de contorno de temperatura constante prescrita no bordo superior

$$\boxed{(i=nr)\quad(j=nc)}\ \rightarrow\ \boxed{u_{nr,nc}=ut}$$

2.3.3.2.2 – Condição de contorno de fluxo de calor radiativo constante prescrito no bordo superior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y\rightarrow qy_{nr,nc}\cong-k\cdot\frac{u_{nr+1,nc}-u_{nr-1,nc}}{2\Delta y}\cdot(+1)=qt\ \rightarrow\ \ u_{nr+1,nc}=u_{nr-1,nc}-qt\cdot\frac{2\Delta y}{k}$$

$$-u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,nc}^{m+2}-u_{nr+1,nc}^{m+2}=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur\ \Rightarrow$$

$$-u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,nc}^{m+2}-\left(u_{nr-1,nc}^{m+2}-qt\cdot\frac{2\Delta y}{k}\right)=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur\ \Rightarrow$$

$$\boxed{(i=nr)\quad(j=nc)}\quad\rightarrow\quad\boxed{-2u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,nc}^{m+2}=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur-qt\cdot\frac{2\Delta y}{k}}$$

2.3.3.2.3 – Condição de contorno de troca de calor por convecção no bordo superior

$$qy=-k\cdot\frac{du}{dy}\cdot n_y=ht\cdot(u-ut)\rightarrow-k\cdot\frac{u_{nr+1,nc}-u_{nr-1,nc}}{2\Delta y}\cdot(+1)=ht\cdot\left(u_{nr,nc}-ut\right)\rightarrow\quad u_{nr+1,nc}=u_{nr-1,nc}-\frac{2\Delta y\cdot ht}{k}\cdot u_{nr,nc}+\frac{2\Delta y\cdot ht}{k}\cdot ut\quad\Rightarrow$$

$$-u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}\right)\cdot u_{nr,nc}^{m+2}-\left(u_{nr-1,nc}^{m+2}-\frac{2\Delta y\cdot ht}{k}\cdot u_{nr,nc}^{m+2}+\frac{2\Delta y\cdot ht}{k}\cdot ut\right)=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur\quad\Rightarrow$$

$$\boxed{(i=nr)\quad(j=nc)}\quad\rightarrow\quad\boxed{-2u_{nr-1,nc}^{m+2}+\left(2+\frac{1}{r_y}+\frac{2\Delta y\cdot ht}{k}\right)\cdot u_{nr,nc}^{m+2}=2f_y\cdot u_{nr,nc-1}^{m+1}+\left(\frac{1}{r_y}-2f_y-f_y\cdot\frac{2\Delta x\cdot hr}{k}\right)\cdot u_{nr,nc}^{m+1}+\frac{s_{nr,nc}^{m+1}}{k\cdot e}\cdot\Delta y^2+f_y\cdot\frac{2\Delta x\cdot hr}{k}\cdot ur+\frac{2\Delta y\cdot ht}{k}\cdot ut}$$