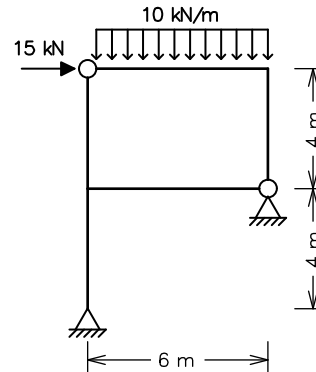


ENG 1204 - ANÁLISE DE ESTRUTURAS II - 1º Semestre - 2010

Prova Final - 07/07/2010 - Duração: 2:30 hs - Sem Consulta

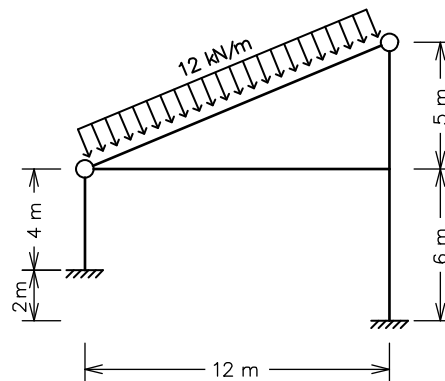
1ª Questão (5,0 pontos)

Determine pelo Método das Forças o diagrama de momentos fletores do quadro hiperestático ao lado. Todas as barras têm a mesma inércia à flexão $EI = 3.6 \times 10^4$ kNm^2 . Somente considere deformações por flexão.



2ª Questão (5,0 pontos)

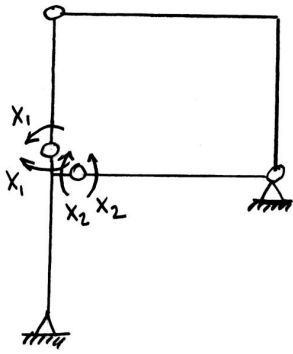
Empregando-se o Método dos Deslocamentos, obter o diagrama de momentos fletores para o quadro ao lado (barras inextensíveis). Todas as barras têm a mesma inércia à flexão $EI = 2.88 \times 10^4$ kNm^2 .



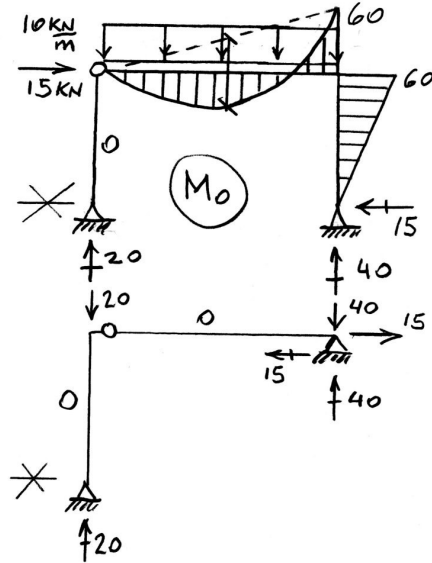
Solução de um sistema de 2 equações a 2 incógnitas:

$$\begin{Bmatrix} e \\ f \end{Bmatrix} + \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} X_1 = \frac{bf - de}{ad - bc} \\ X_2 = \frac{ce - af}{ad - bc} \end{cases}$$

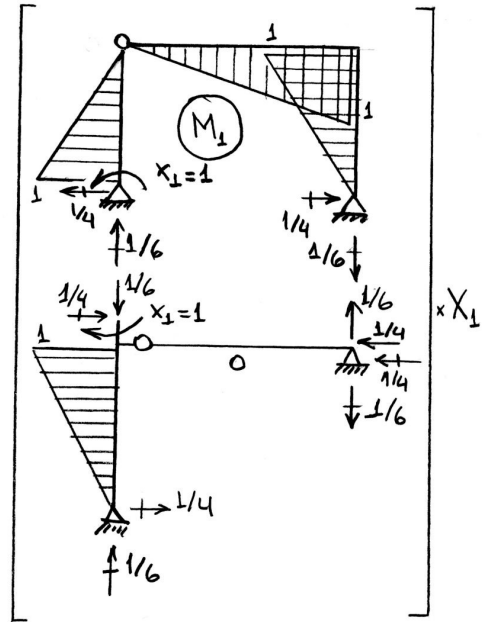
1ª Questão
Sistema Principal e
Hiperestáticos



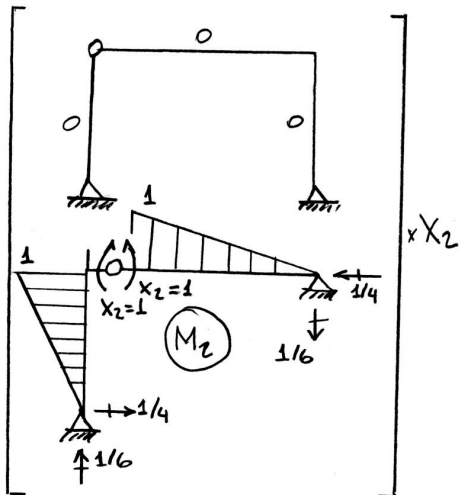
(0) Solicitação externa
isolada no S.P.



(1) X_1 isolado no S.P.



(2) X_2 isolado no S.P.



Eqns de Compatibilidade

$$\begin{cases} \delta_{10} + \delta_{11} X_1 + \delta_{12} X_2 = 0 \\ \delta_{20} + \delta_{21} X_1 + \delta_{22} X_2 = 0 \end{cases}$$

$$EI \delta_{10} = -\frac{1}{3} \times 6 \times 60 \times 1 + \frac{1}{3} \times 6 \times 45 \times 1 - \frac{1}{3} \times 4 \times 60 \times 1 = -110$$

$$EI \delta_{20} = 0$$

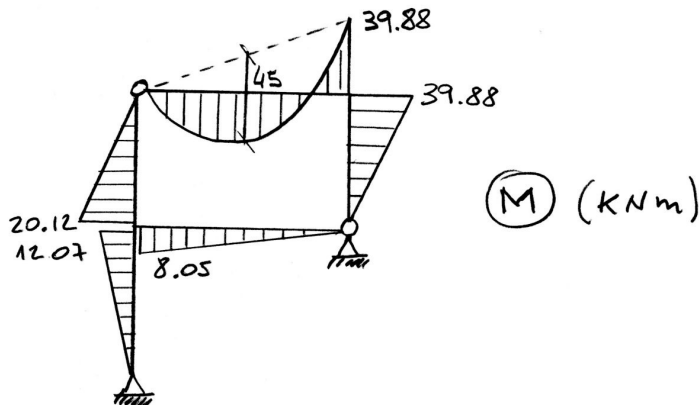
$$EI \delta_{11} = 3 \times \frac{1}{3} \times 4 \times 1 \times 1 + \frac{1}{3} \times 6 \times 1 \times 1 = 6$$

$$EI \delta_{12} = EI \delta_{21} = +\frac{1}{3} \times 4 \times 1 \times 1 = 4/3$$

$$EI \delta_{22} = \frac{1}{3} \times 4 \times 1 \times 1 + \frac{1}{3} \times 6 \times 1 \times 1 = 10/3$$

$$\Rightarrow \frac{1}{EI} \begin{bmatrix} 6 & 4/3 \\ 4/3 & 10/3 \end{bmatrix} \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} = \frac{1}{EI} \begin{Bmatrix} 110 \\ 0 \end{Bmatrix} \Rightarrow \begin{cases} X_1 = 20.12 \text{ kNm} \\ X_2 = -8.05 \text{ kNm} \end{cases}$$

Momentos Fletores Finais: $M = M_0 + M_1 X_1 + M_2 X_2$

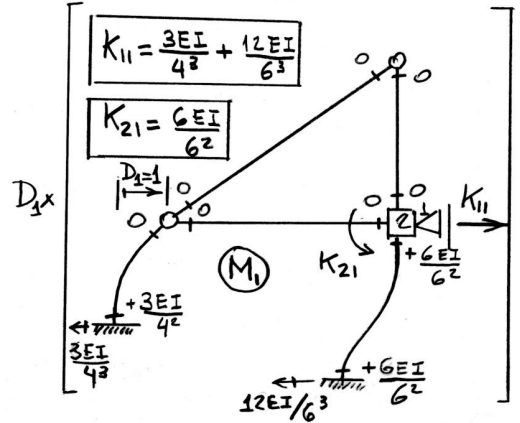
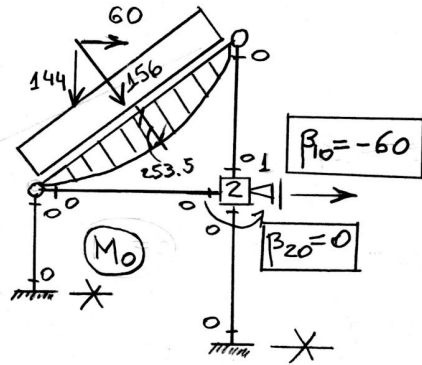
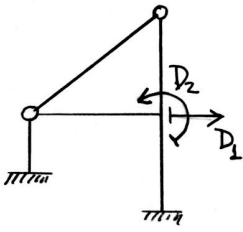


2ª Questão

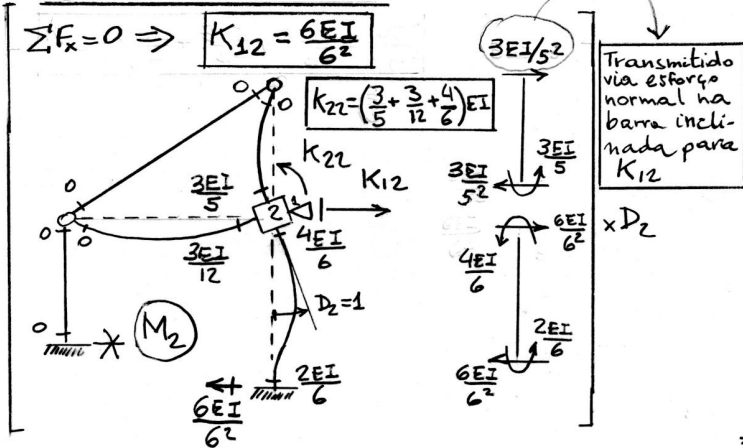
(0) Solicitação externa
no S.H. ($D_1=0, D_2=0$)

(1) D_1 isolado no S.H.

Deslocabilidades:



(2) D_2 isolado no S.H.



Sistema de Eqs. de Equilíbrio

$$\begin{cases} \beta_{10} + K_{11}D_1 + K_{12}D_2 = 0 \\ \beta_{20} + K_{21}D_1 + K_{22}D_2 = 0 \end{cases}$$

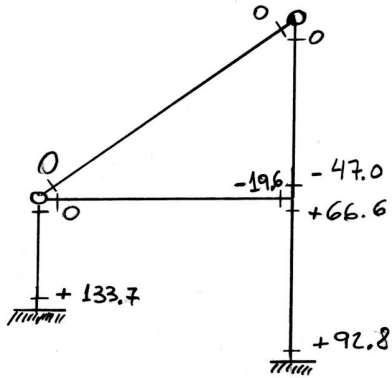
$$EI = 28800 \text{ kNm}^2 \Rightarrow$$

$$\begin{bmatrix} 2950 & 4800 \\ 4800 & 43680 \end{bmatrix} \begin{Bmatrix} D_1 \\ D_2 \end{Bmatrix} = \begin{Bmatrix} 60 \\ 0 \end{Bmatrix}$$

$$\Rightarrow D_1 = 2.48 \times 10^{-2} \text{ m} \quad D_2 = -2.72 \times 10^{-3} \text{ rad}$$

Momentos Fletores Finais

$$M = M_0 + M_1 D_1 + M_2 D_2$$



(M)
(kNm)

